Pre-calculus
Trig Ch. 5/6 Review

Name: Key
Period: ______

Write the complex number in standard form:

1. $\sqrt{-64}$

$\boxed{8i \text{ or } 0+8i}$

2. $\sqrt{25}+\sqrt{-9}$

$\boxed{5+3i}$

3. $11+\sqrt{-48}$

$\boxed{11+4i\sqrt{3}}$

Simplify and write the complex number in standard form:

3. $(4-8i)+(5+3i)$

$\boxed{9-5i}$

5. $(-2-4i)-(5-8i)$

$\boxed{-7+4i}$

6. $\sqrt{-12}\cdot\sqrt{-27}$

$2i\sqrt{3}\cdot 3i\sqrt{3}$

$= \boxed{-18 \text{ or } -18+0i}$

7. $3i(2+5i)+2i(3-4i)$

$6i+15i^2+6i-8i^2$

$\boxed{-7+12i}$

8. $3-(4-3i)$

$\boxed{-1+3i}$

9. $(6+5i)(2-5i)$

$12-30i+10i-25i^2$

$\boxed{37-20i}$

10. $\frac{-8}{2i}\cdot\frac{i}{i}=\frac{-8i}{2i^2}$

$\boxed{4i \text{ or } 0+4i}$

11. $\frac{5}{3+4i}\quad\frac{3-4i}{3-4i}$

$\frac{15-20i}{9-16i^2}=\frac{15-20i}{25}$

$=\boxed{\frac{3}{5}-\frac{4}{5}i}$

12. $\frac{4+i}{3+5i}\quad\frac{3-5i}{3-5i}$

$\frac{12-17i-5i^2}{9-25i^2}=\frac{17-17i}{34}$

$=\boxed{\frac{1}{2}-\frac{1}{2}i}$

Evaluate the powers of i:

13. i^{66}

$i^{64}\cdot i^2=1\cdot -1=\boxed{-1}$

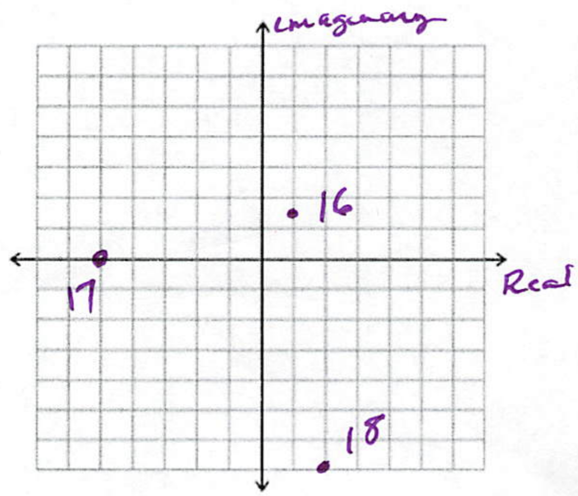
14. $-i^{51}$

$-i^{48}\cdot i^3=-(1)(-i)=\boxed{i}$

15. i^{-52}

$\frac{1}{i^{52}}=\frac{1}{1}=\boxed{1}$

Graph each complex number and find it's **absolute value:**



16. $z=1+i\sqrt{3}$

$|z|=\sqrt{1^2+\sqrt{3}^2}=\boxed{2}$

17. $z=-5$

$|z|=\boxed{5}$

18. $z=2-7i$

$|z|=\sqrt{2^2+(-7)^2}=\boxed{\sqrt{53}}$

Write each complex number in trigonometric form using **radians** (in terms of π):

19. $z=-4-4i$

$r=\sqrt{(-4)^2+(-4)^2}=4\sqrt{2}$

$\tan\theta=\frac{-4}{-4}\quad\theta=\frac{5\pi}{4}$

$\boxed{4\sqrt{2}\text{ cis }\frac{5\pi}{4}}$

20. $-2i$

$r=2$

$\theta=\frac{3\pi}{2}$

$\boxed{2\text{ cis }\frac{3\pi}{2}}$

21. $1+i\sqrt{3}$

$r=\sqrt{1^2+(\sqrt{3})^2}=2$

$\tan\theta=\frac{\sqrt{3}}{1}\quad\theta=\frac{\pi}{3}$

$\boxed{2\text{ cis }\frac{\pi}{3}}$

Write each complex number in **standard form**:

22. $z = 6(\cos 210° + i\sin 210°)$

$6\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$\boxed{= -3\sqrt{3} - 3i}$

23. $z = 6\ cis\ 135°$

$6(\cos 135 + i \sin 135)$

$= 6\left(-\frac{\sqrt{2}}{2} + i\ \frac{\sqrt{2}}{2}\right)$

$= \boxed{-3\sqrt{2} + 3\sqrt{2}i}$

24. $z = 9\ cis\ \frac{4\pi}{3}$

$9\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$9\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) =$

$\boxed{-\frac{9}{2} - \frac{9\sqrt{3}}{2}i}$

25. $z = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$4\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\boxed{= 2 - 2\sqrt{3}i}$

Multiply or divide the complex numbers and write your answers in **standard form:**

26. $4\ cis\ 120° \cdot 6\ cis\ 120°$

$24\ cis\ 240°$

$24\left(-\frac{1}{2} + i - \frac{\sqrt{3}}{2}\right) = \boxed{-12 - 12i\sqrt{3}}$

27. $\frac{15 cis\ 240°}{3\ cis\ 135°} = 5\ cis\ 105°$

$= 5(\cos 105° + i\sin 105°)$

$\boxed{= -1.29 + 4.83i}$

28. $8(\cos 76° + i\sin 76°) \cdot 7(\cos 144° + i\sin 144°)$

$56\left(\cos \overset{220}{\cancel{200}} + i\sin \overset{220}{\cancel{200}}\right)$

$= \boxed{\cancel{-90.21 - 32.83i}} = \boxed{-42.9 - 36i}$

29. $\frac{9(\cos 25° + i\sin 25°)}{3(\cos 175° + i\sin 175°)} = 3\ cis(-150°) = 3\ cis\ 210°$

$= 3(\cos 210° + i\sin 210°) = 3\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$= \boxed{-\frac{3\sqrt{3}}{2} - \frac{3}{2}i}$

30. Find all indicated powers. Write all answers in **standard form.**

A. $(\cos 240° + i\sin 240°)^{12}$

$1^{12}\ cis\ (12 \cdot 240) = cis\ 2880°$

$= cis\ 0° = \boxed{1 \text{ or } 1 + 0i}$

B. $(2\ cis\ 330°)^4$

$2^4 cis(330 \cdot 4) = 16\ cis\ 1320°$

$16\ cis\ 240° = 16\left(-\frac{1}{2} + i\ \frac{-\sqrt{3}}{2}\right)$

$= \boxed{-8 - 8\sqrt{3}i}$

C. $(2\sqrt{3} - 2i)^5$

$r = \sqrt{(2\sqrt{3})^2 + (-2)^2} = 4$

$\tan\theta = \frac{-2}{2\sqrt{3}}$ $\theta = 330°$

$(4\ cis\ 330)^5 = 4^5 cis\ 5 \cdot 330 = 1024\ cis\ 1650$

$= 1024\ cis\ 210 = 1024\left(-\frac{\sqrt{3}}{2} + i \cdot \frac{-1}{2}\right) = \boxed{-512\sqrt{3} - 512i}$ $\cancel{-512 - 512\sqrt{3}}$

D. $\left(-\frac{\sqrt{2}}{2} + i\ \frac{\sqrt{2}}{2}\right)^{12}$

$r = 1$ $(1\ cis\ 135)^{12}$

$\theta = 135$ $= 1^{12} cis(12 \cdot 135)$

$1\ cis\ 1620 = cis\ 180$

$\boxed{-1 \text{ or } -1 + 0i}$

30. Find the indicated roots. Write all answers in **standard form.**

A. The four fourth roots of -16

$r = 16$

$\theta = 180^\circ$

$n = 4$

$\sqrt[4]{16}\ \text{cis}\ \dfrac{180 + 360(0)}{4}$

$k=0 \quad 2\ \text{cis}\ 45 = \sqrt{2} + \sqrt{2}i$

$k=1 \quad 2\ \text{cis}\ 135 = -\sqrt{2} + \sqrt{2}i$

$k=2 \quad 2\ \text{cis}\ 225 = -\sqrt{2} - \sqrt{2}i$

$k=3 \quad 2\ \text{cis}\ 315 = \sqrt{2} - \sqrt{2}i$

B. The three cube roots of i.

$r = 1$

$\theta = 90^\circ$

$n = 3$

$\sqrt[3]{1}\ \text{cis}\ \dfrac{90 + 360(0)}{3}$

$k=0 \quad \text{cis}\ 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k=1 \quad \text{cis}\ 150 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

$k=2 \quad \text{cis}\ 270^\circ = 0 - i$

C. The five fifth roots of $-1 + i$.

$r = \sqrt{2}$

$\theta = 135^\circ$

$n = 5$

$\sqrt[5]{\sqrt{2}}\ \text{cis}\ \dfrac{135 + 360(0)}{5}$

$k=0 \quad 2^{1/10}\ \text{cis}\ 27^\circ = .953 + .49i$

$k=1 \quad 1.07\ \text{cis}\ 99^\circ = -.17 + 1.06i$

$k=2 \quad 1.07\ \text{cis}\ 171^\circ = -1.06 + .17i$

$k=3 \quad 1.07\ \text{cis}\ 243^\circ = -.49 - .95i$

$k=4 \quad 1.07\ \text{cis}\ 315^\circ = .76 - .76i$

31. Convert each of the following points from polar to rectangular coordinates.

A. $(2, \pi)$ $(-2, 0)$

$x = 2\cos\pi = -2$

$y = 2\sin\pi = 0$

B. $\left(4, \frac{\pi}{4}\right)$ $(2\sqrt{2}, 2\sqrt{2})$

$x = 4\cos\frac{\pi}{4} = 2\sqrt{2}$

$y = 4\sin\frac{\pi}{4} = 2\sqrt{2}$

C. $\left(1, \frac{\pi}{6}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$x = 1\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$y = 1\sin\frac{\pi}{6} = \frac{1}{2}$

32. Convert each of the following points from rectangular to polar coordinates:
(Use **radians** for θ)

A. $(-2\sqrt{3}, 2)$ $\left(4, \frac{5\pi}{6}\right)$

$r = \sqrt{(-2\sqrt{3})^2 + 2^2} = 4$

$\tan\theta = \frac{2}{-2\sqrt{3}} \quad \theta = \frac{5\pi}{6}$

B. $(8, -15)$ $(17, 298.1^\circ)$

$\theta = 298.1$

$r = \sqrt{8^2 + (-15)^2}$

$= 17$

$\tan\theta = \frac{-15}{8}$

$\theta = -61.9$

C. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\left(1, \frac{7\pi}{4}\right)$

$r = 1$

$\theta = \frac{7\pi}{4}$

33. Find a rectangular form of each of the polar equations.

A. $r = -4\sin\theta$

$r^2 = -4r\sin\theta$

$x^2 + y^2 = -4y$

$x^2 + y^2 + 4y = 0$

$x^2 + y^2 + 4y + 4 = 4$

$x^2 + (y+2)^2 = 4$

B. $r = 2\csc\theta$

$r = \frac{2}{\sin\theta}$

$r\sin\theta = 2$

$y = 2$

C. $r(2\cos\theta + \cancel{3}\sin\theta) = 8$

$2r\cos\theta + \cancel{3}r\sin\theta = 8$

$2x + y = 8$

34. Find a polar form of each equation.

A. $y = -6$

$r \sin\theta = -6$

$r = \frac{-6}{\sin\theta}$

$\boxed{r = -6\csc\theta}$

B. $3x + 4y = 9$

$3r\cos\theta + 4r\sin\theta = 9$

$r(3\cos\theta + 4\sin\theta) = 9$

$\boxed{r = \frac{9}{3\cos\theta + 4\sin\theta}}$

C. $x^2 + y^2 = 6y$

$r^2 = 6r\sin\theta$

$\boxed{r = 6\sin\theta}$

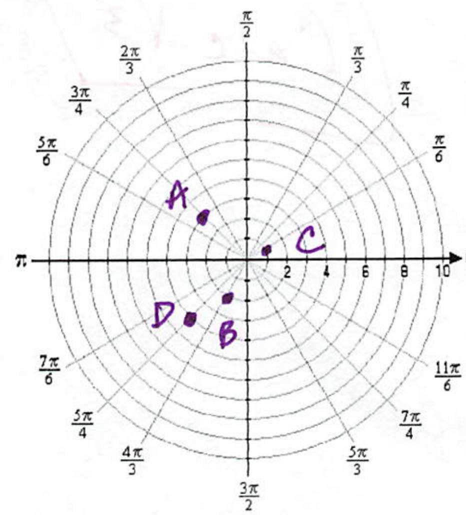
35. Plot the coordinate on the polar coordinate system.

A. $\left(3, \frac{3\pi}{4}\right)$

B. $\left(-2, \frac{\pi}{3}\right)$

C. $\left(-1, -\frac{5\pi}{6}\right)$

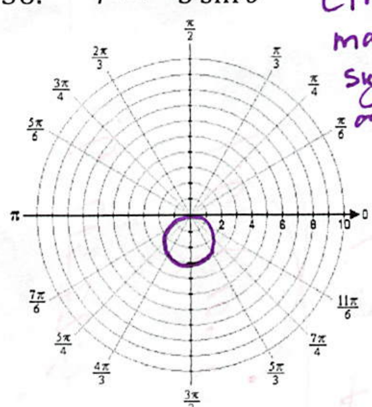
D. $\left(4, -\frac{3\pi}{4}\right)$



Sketch the graph of each polar equation. Identify the type and any key features:

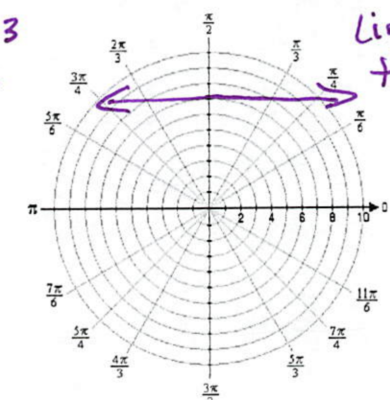
36. $r = -3\sin\theta$

Circle
max r = 3
Sym on $\theta = \frac{\pi}{2}$



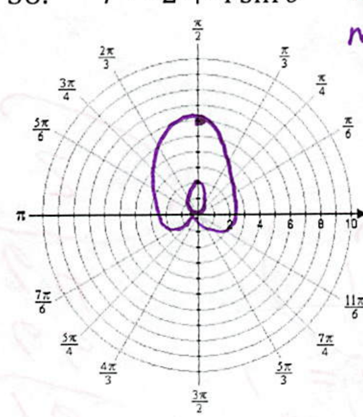
37. $r = 7\csc\theta$

Line
Hor.



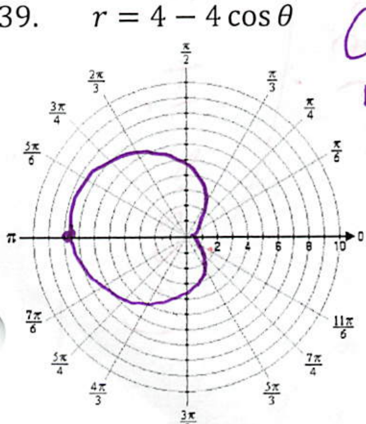
38. $r = 2 + 4\sin\theta$

Limacon
max r = 6
loop = 2
Sym on $\theta = \frac{\pi}{2}$



39. $r = 4 - 4\cos\theta$

Cardioid
max r = 8
flipped
Sym on polar axis



40. $r = 5\cos 3\theta$

Rose
3 petals
max r = 5
Sym on polar axis