

Name _____

Trigonometry Review Chapter 21. Let θ be an acute angle of a right triangle and $\sec \theta = \frac{9}{5}$

a) $\sin \theta$

Answer a) _____

b) $\tan \theta$

Answer b) _____

2. Let θ be an acute angle of a right triangle and $\cos \theta = \frac{7}{11}$

a) $\cot \theta$

Answer a) _____

b) $\csc \theta$

Answer b) _____

3. Find the supplement and complement of each angle measurement:

a) 98°

Answer a) _____

b) $\frac{3\pi}{7}$

Answer b) _____

c) $59^\circ 17' 32''$

Answer c) _____

4. Convert from degrees to radians

a) 160°

Answer a) _____

b) -380°

Answer b) _____

5. Convert from radians to degrees

a) $\frac{2\pi}{5}$

Answer a) _____

b) $-\frac{14\pi}{9}$

Answer b) _____

6. Find the measure of two angles (one positive and one negative) that are co-terminal with the given angle measurement

a) $-\frac{3\pi}{7}$

Answer a) _____

b) 170°

Answer b) _____

7. Given $\csc \alpha = -\frac{2\sqrt{3}}{3}$, and $180^\circ < \alpha < 270^\circ$. Find the **exact** value of:

a) $\tan \alpha$

Answer a) _____

b) $\cos \alpha$

Answer b) _____

8. Given $\cot \beta = -\frac{\sqrt{3}}{3}$, and $\frac{3\pi}{2} < \beta < 2\pi$. Find the **exact** value of:

a) $\sin \beta$

Answer a) _____

b) $\sec \beta$

Answer b) _____

9. Find the **exact** measure of the intercepted arc of a circle with the given radius and central angle: ($s = r\theta$)

a) $r = 16 \text{ ft}; \theta = \frac{3\pi}{8}$

Answer a) _____

b) $r = 8 \text{ cm}; \theta = 75^\circ$

Answer b) _____

10. A parasail rider is attached to a boat with a rope that is 80 feet long. The angle of elevation from the boat to the parasail rider is 36° .

a) Draw and label a right triangle to represent the situation.

b) Write and solve a trigonometric function that can be used to estimate the parasail rider's height above the boat. Round your answer to the nearest tenth.

Answer _____

11. An observer on the ground at point A watches a rocket ascend 1800 feet. The observer is 1200 feet from the launch point B.

a) Draw and label a right triangle to represent the situation.

b) Write and solve a trigonometric function that can be used to find the grade (i.e., angle of elevation)? Round your answer to the nearest degree.

Answer _____

c) How far from the rocket is the observer? Round your answer to the nearest foot.

Answer _____

12. Show **algebraically** whether the following functions are odd, even or neither.

a. $f(x) = -3 \sin x \cos x$

Answer a) _____

b. $f(x) = \csc x \cot x$

Answer b) _____

c. $f(x) = x \cos x$

Answer c) _____

d. $f(x) = \sec x + \sin x$

Answer d) _____

Do not use a calculator on this part of the test

13. Find the simplified exact value of each of the following expressions:

a. $\csc(-540^\circ)$

Answer a) _____

b. $\sin\left(-\frac{4\pi}{3}\right)$

Answer b) _____

c. $\csc\frac{5\pi}{3}$

Answer c) _____

d. $\cos(1200^\circ)$

Answer d) _____

e. $\sin\left(-\frac{7\pi}{6}\right)$

Answer e) _____

f. $\sec\frac{8\pi}{3}$

Answer f) _____

g. $\csc\frac{3\pi}{4} + \sin\frac{3\pi}{2}$

Answer g) _____

h. $\tan^2\frac{2\pi}{3} + \cot^2\frac{\pi}{2}$

Answer h) _____

i. $\sec^2\frac{\pi}{3} - \tan^2\frac{\pi}{3}$

Answer i) _____

j. $\tan\frac{\pi}{3} - \cos\frac{\pi}{6}$

Answer j) _____

14. Simplify the following expressions to a single trigonometric function.

a. $\sin \theta \tan \theta \cot \theta \csc \theta$

Answer a) _____

b. $\frac{\sec^2 \theta - 1}{\sec^2 \theta}$

Answer b) _____

c. $\cos \theta \sec \theta - \frac{\cos \theta}{\sec \theta}$

Answer c) _____

d. $\frac{\sin \theta + \tan \theta}{1 + \sec \theta}$

Answer d) _____

e. $\frac{\sin \theta}{1 - \cos \theta} + \frac{1 - \cos \theta}{\sin \theta}$

Answer e) _____

f. $\frac{\sec \theta - \cos \theta}{\tan \theta \sin \theta}$

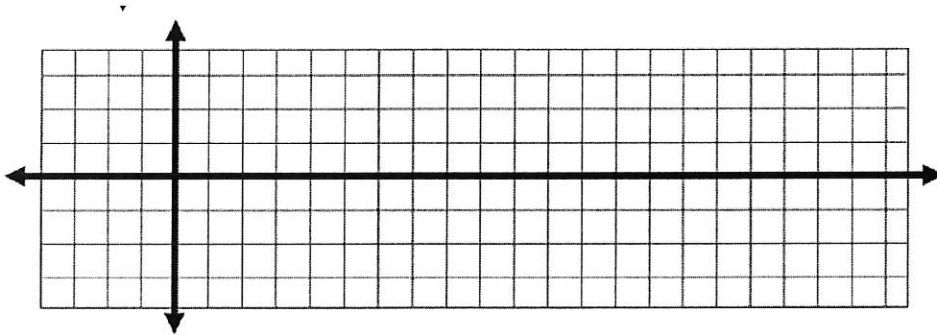
Answer f) _____

For each of the following functions identify key features, **amplitude, period, phase shift and vertical shift, asymptotes and zeros** if applicable. Then graph one periods (in radians).

Label the critical points and the x and y-axes well.

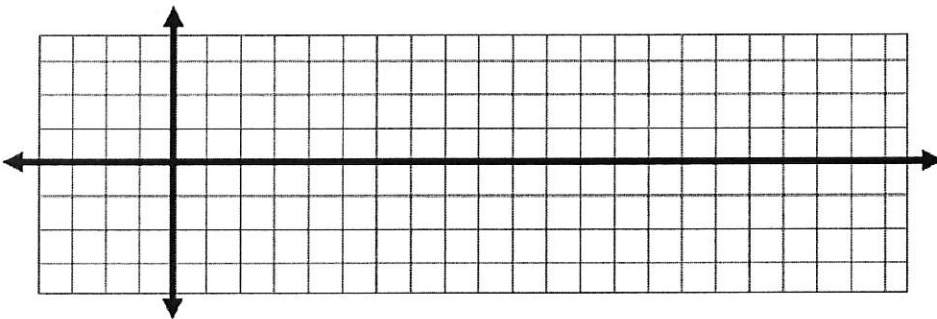
15. $f(x) = \sin\left(3\theta + \frac{3\pi}{2}\right) - 2$

Key Features:



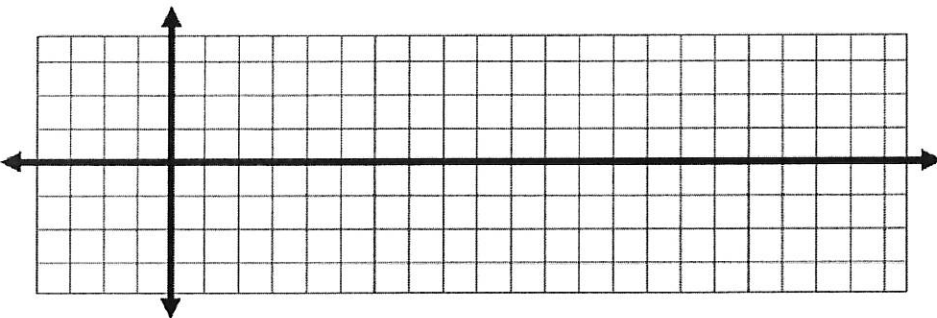
16. $f(x) = 2\cos\left(\theta - \frac{\pi}{3}\right)$

Key Features:



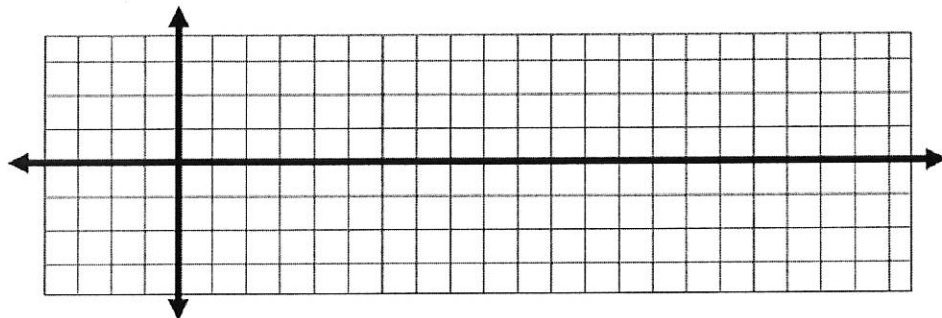
17. $f(x) = \tan\left(\theta - \frac{\pi}{4}\right) + 2$

Key Features:



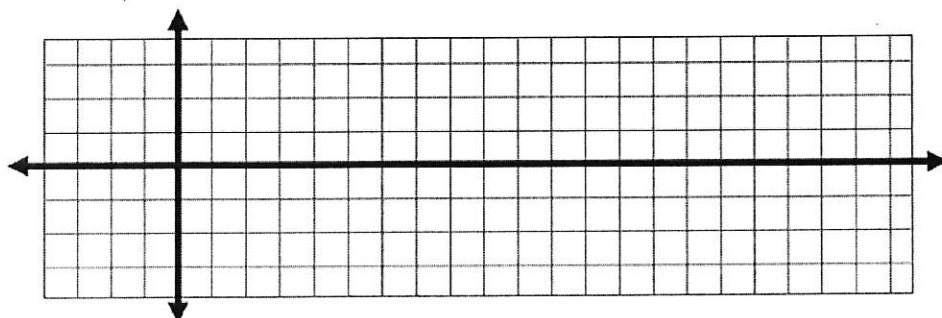
$$18. f(x) = \cot 2\left(\theta + \frac{\pi}{2}\right) - 1$$

Key Features:



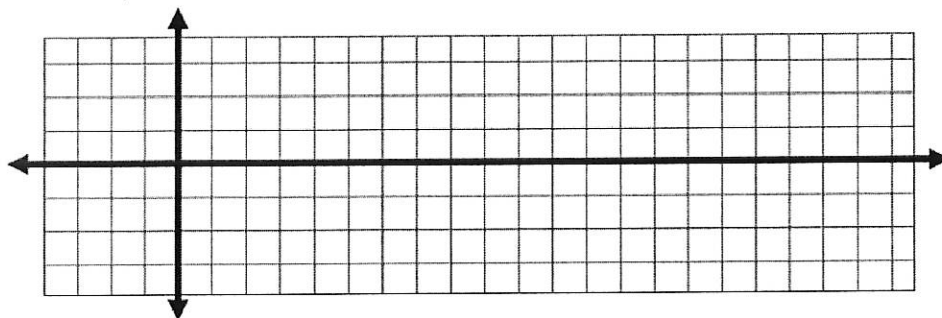
$$19. f(x) = -3\sec\left(\frac{1}{2}\theta\right) + 2$$

Key Features:



$$20. f(x) = \csc\left(\frac{1}{4}\theta + \pi\right) - 3$$

Key Features:

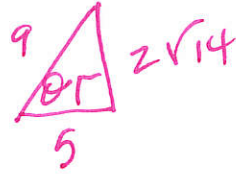


Name Key

Trigonometry Review Chapter 2

1. Let θ be an acute angle of a right triangle and $\sec \theta = \frac{9}{5}$

a) $\sin \theta$



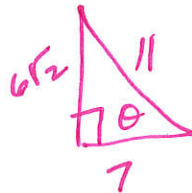
b) $\tan \theta$

Answer a) $\frac{2\sqrt{14}}{9}$

Answer b) $\frac{2\sqrt{14}}{5}$

2. Let θ be an acute angle of a right triangle and $\cos \theta = \frac{7}{11}$

a) $\cot \theta$



b) $\csc \theta$

Answer a) $\frac{7}{6\sqrt{2}} = \frac{7\sqrt{2}}{12}$

Answer b) $\frac{11}{6\sqrt{2}} = \frac{11\sqrt{2}}{12}$

3. Find the supplement and complement of each angle measurement:

a) 98°

b) $\frac{3\pi}{7}$

c) $59^\circ 17' 32''$
 $\underline{30^\circ 42' 28''}$

Answer a) c: none
s: 82°

Answer b) c: $\frac{\pi}{14}$
s: $4\pi/7$

Answer c) $30^\circ 42' 28''$

4. Convert from degrees to radians

a) 160°

b) -380°

Answer a) $\frac{8\pi}{9}$

Answer b) $-\frac{19\pi}{9}$

5. Convert from radians to degrees

a) $\frac{2\pi}{5}$

b) $-\frac{14\pi}{9}$

Answer a) 72°

Answer b) -280°

6. Find the measure of two angles (one positive and one negative) that are coterminal with the given angle measurement

a) $-\frac{3\pi}{7} \pm \frac{14\pi}{7}$

Answer a) $\frac{11\pi}{7}, -\frac{17\pi}{7}$

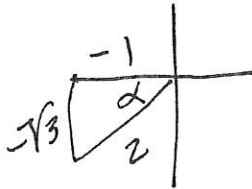
b) $170^\circ \pm 360$

Answer b) $530^\circ, -190^\circ$

7. Given $\csc \alpha = -\frac{2\sqrt{3}}{3}$, and $180^\circ < \alpha < 270^\circ$. Find the exact value of:

$\sin \alpha = \frac{3}{2\sqrt{3}}$

a) $\tan \alpha$



Answer a) $\sqrt{3}$

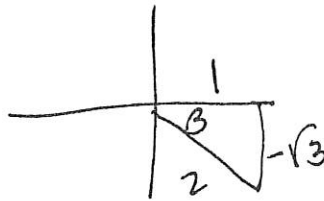
$\frac{3\sqrt{3}}{2 \cdot 3}$

b) $\cos \alpha$

Answer b) $-\frac{1}{2}$

8. Given $\cot \beta = -\frac{\sqrt{3}}{3}$, and $\frac{3\pi}{2} < \beta < 2\pi$. Find the exact value of:

a) $\sin \beta$



Answer a) $-\frac{\sqrt{3}}{2}$

$\frac{3\sqrt{3}}{\sqrt{3} \cdot 3}$

b) $\sec \beta$

Answer b) 2

9. Find the exact measure of the intercepted arc of a circle with the given radius and central angle:

a) $r = 16 \text{ ft}; \theta = \frac{3\pi}{8}$

$16 \cdot \frac{3\pi}{8}$

$S = r \cdot \theta$

Answer a) $6\pi \text{ ft.}$

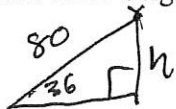
b) $r = 8 \text{ cm}; \theta = 75^\circ$

$S = 8 \cdot 75 \cdot \frac{\pi}{180}$

Answer b) $\frac{10\pi}{3} \text{ cm}$

10. A parasail rider is attached to a boat with a rope that is 80 feet long. The angle of elevation from the boat to the parasail rider is 36° .

a) Draw and label a right triangle to represent the situation.



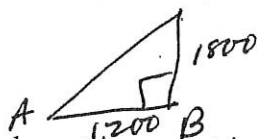
b) Write and solve a trigonometric function that can be used to estimate the parasail rider's height above the boat. Round your answer to the nearest tenth.

$$\sin 36^\circ = \frac{h}{80}$$

Answer 47.0 ft.

11. An observer on the ground at point A watches a rocket ascend 1800 feet. The observer is 1200 feet from the launch point B.

a) Draw and label a right triangle to represent the situation.



b) Write and solve a trigonometric function that can be used to find the grade (i.e., angle of elevation)? Round your answer to the nearest degree.

$$\tan A = \frac{1800}{1200}$$

Answer 56°

c) How far from the rocket is the observer? Round your answer to the nearest foot.

$$\sqrt{1200^2 + 1800^2}$$

Answer 2163 ft.

12. Show algebraically whether the following functions are odd, even or neither.

a. $f(x) = -3 \sin x \cos x$

$$f(-x) = -3 \sin(-x) \cos(-x)$$

Answer a) odd

b. $f(x) = \csc x \cot x$

$$= 3 \sin x \cos x$$

$$f(-x) = \csc(-x) \cot(-x)$$

$$= -\csc x (-\cot x) = \csc x \cot x$$

Answer b) even

c. $f(x) = x \cos x$

$$f(-x) = -x \cos(-x)$$

$$= -x \cos x$$

Answer c) odd

d. $f(x) = \sec x + \sin x$

$$f(-x) = \sec(-x) + \sin(-x)$$

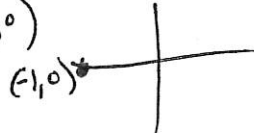
$$\sec x - \sin x$$

Answer d) neither

Do not use a calculator on this part of the test

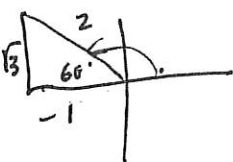
13. Find the simplified exact value of each of the following expressions:

a. $\csc(-540^\circ) = \csc(180^\circ)$



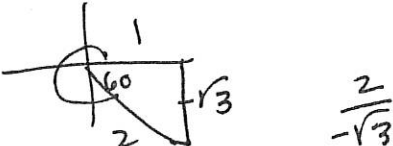
Answer a) undefined

b. $\sin(-\frac{4\pi}{3}) = \sin(\frac{2\pi}{3})$




Answer b) $\frac{\sqrt{3}}{2}$

c. $\csc\frac{5\pi}{3}$



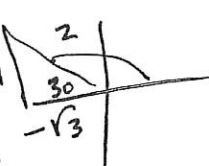
Answer c) $-\frac{2\sqrt{3}}{3}$

d. $\cos(1200^\circ) = \cos(120^\circ)$



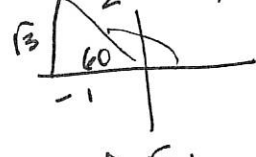
Answer d) $-\frac{1}{2}$

e. $\sin(-\frac{7\pi}{6}) = \sin\frac{5\pi}{6}$




Answer e) $\frac{1}{2}$

f. $\sec\frac{8\pi}{3} = \sec\frac{2\pi}{3}$



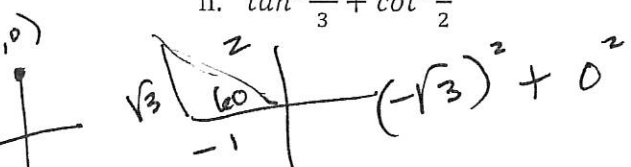
Answer f) -2

g. $\csc\frac{3\pi}{4} + \sin\frac{3\pi}{2} = -\sqrt{2} + -1$



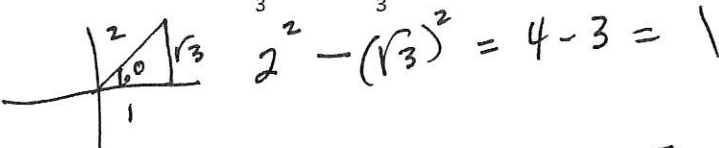
Answer g) $\sqrt{2} - 1$

h. $\tan^2\frac{2\pi}{3} + \cot^2\frac{\pi}{2} = (-\sqrt{3})^2 + 0^2$



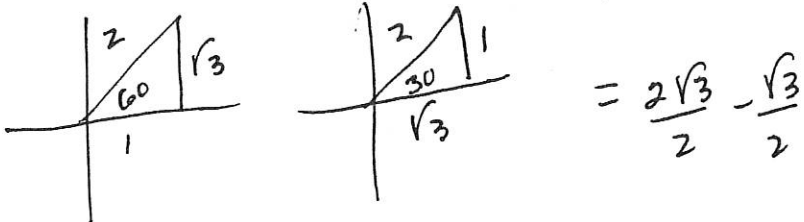
Answer h) 3

i. $\sec^2\frac{\pi}{3} - \tan^2\frac{\pi}{3} = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$



Answer i) 1

j. $\tan\frac{\pi}{3} - \cos\frac{\pi}{6} = \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$



Answer j) $\frac{\sqrt{3}}{2}$

14. Simplify the following expressions to a single trigonometric function.

a. $\sin \theta \tan \theta \cot \theta \csc \theta$

$$\sin \theta \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta}$$

Answer a) 1

b. $\frac{\sec^2 \theta - 1}{\sec^2 \theta}$

$$\frac{\tan^2 \theta}{\sec^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\frac{1}{\cos^2 \theta}} = \sin^2 \theta$$

Answer b) $\sin^2 \theta$

c. $\cos \theta \sec \theta - \frac{\cos \theta}{\sec \theta}$

$$\cos \theta \cdot \frac{1}{\cos \theta} - \frac{\cos \theta}{\frac{1}{\cos \theta}} = 1 - \cos^2 \theta = \sin^2 \theta$$

Answer c) $\sin^2 \theta$

d. $\frac{\sin \theta + \tan \theta}{1 + \sec \theta}$

$$\begin{aligned} &= \frac{\sin \theta + \frac{\sin \theta}{\cos \theta}}{1 + \frac{1}{\cos \theta}} = \frac{\frac{\sin \theta \cos \theta + \sin \theta}{\cos \theta}}{\frac{\cos \theta + 1}{\cos \theta}} = \frac{\sin \theta \cos \theta + \sin \theta}{\cos \theta + 1} \\ &= \frac{\sin \theta (\cos \theta + 1)}{\cos \theta + 1} = \sin \theta \end{aligned}$$

Answer d) $\sin \theta$

e. $\frac{\sin \theta}{1 - \cos \theta} + \frac{1 - \cos \theta}{\sin \theta}$

$$\begin{aligned} &\frac{\sin \theta \cdot \sin \theta}{\sin \theta (1 - \cos \theta)} + \frac{(1 - \cos \theta)(1 - \cos \theta)}{\sin \theta (1 - \cos \theta)} = \frac{\sin^2 \theta + 1 - 2 \cos \theta + \cos^2 \theta}{\sin \theta (1 - \cos \theta)} \\ &= \frac{2 - 2 \cos \theta}{\sin \theta (1 - \cos \theta)} = \frac{2(1 - \cos \theta)}{\sin \theta (1 - \cos \theta)} \end{aligned}$$

Answer e) $\frac{2}{\sin \theta} = 2 \csc \theta$

f. $\frac{\sec \theta - \cos \theta}{\tan \theta \sin \theta}$

$$\frac{\frac{1}{\cos \theta} - \cos \theta}{\frac{\sin \theta}{\cos \theta} \cdot \sin \theta} = \frac{\frac{1 - \cos^2 \theta}{\cos \theta}}{\frac{\sin^2 \theta}{\cos \theta}} = \frac{1 - \cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta} = 1$$

Answer f) 1

For each of the following functions identify key features, **amplitude, period, phase shift and vertical shift, asymptotes and zeros** if applicable. Then graph two periods (in radians). Label the critical points and the x and y-axes well.

15. $f(x) = \sin\left(3\theta + \frac{3\pi}{2}\right) - 2$

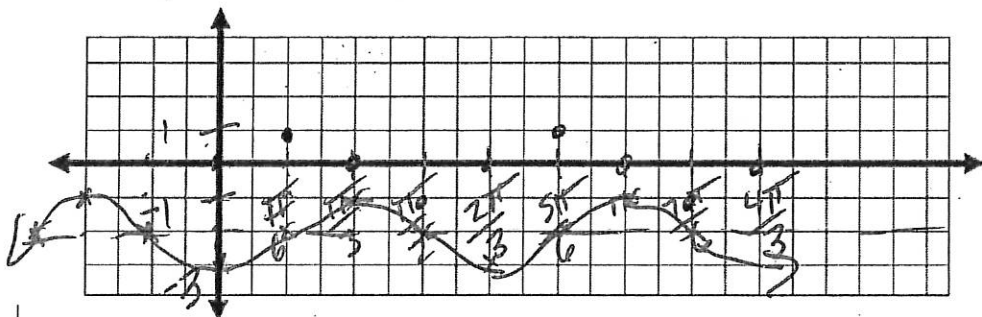
Key Features:

amp: 1
Period: $2\pi/3$

Phase shift: $\pi/2$ left
v. shift: down 2

Critical points

$\frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$



16. $f(x) = 2\cos\left(\theta - \frac{\pi}{3}\right)$

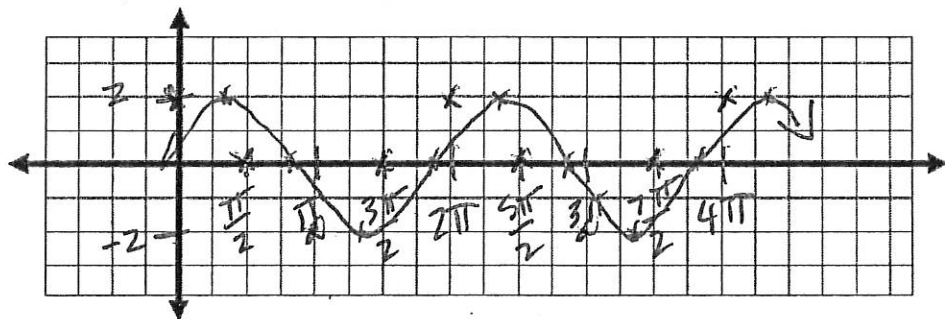
Key Features:

amp: 2
Period: 2π

Phase shift: $\pi/3$ right
v. shift: none

Critical points

2π

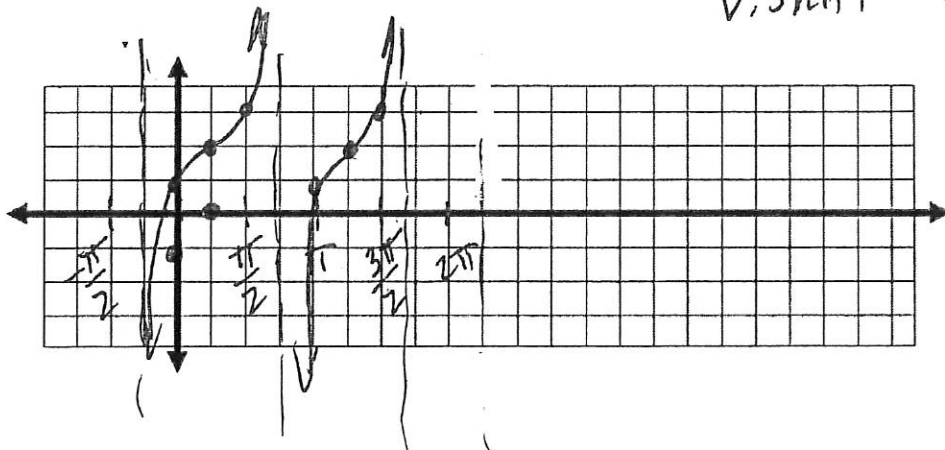


17. $f(x) = \tan\left(\theta - \frac{\pi}{4}\right) + 2$

Key Features:

Period: π

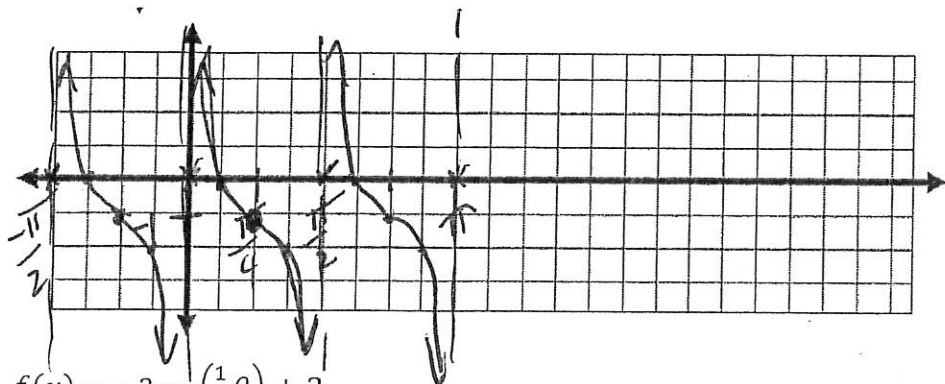
Phase shift: $\pi/4$ right
v. shift up 2



18. $f(x) = \cot 2\left(\theta + \frac{\pi}{2}\right) - 1$

Key Features:

Period: $\frac{\pi}{2}$ p.s. $\frac{\pi}{2}$ left v.s. down 1

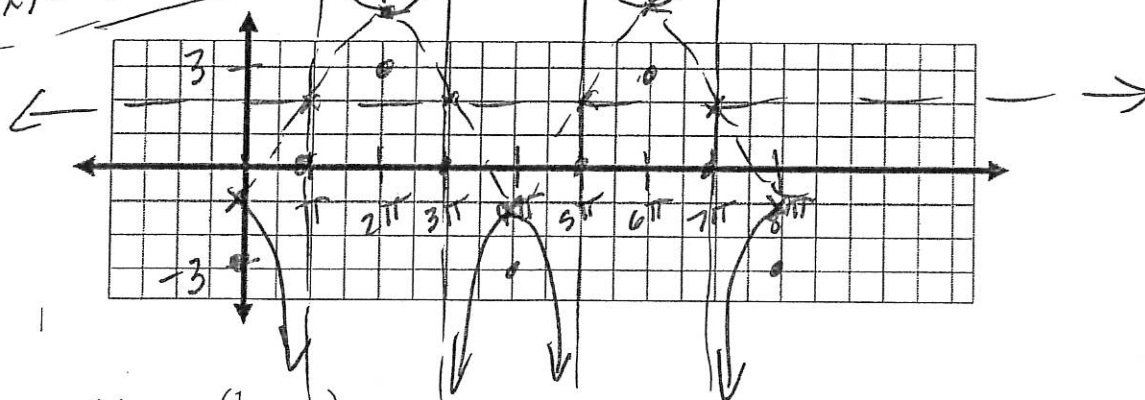


19. $f(x) = -3\sec\left(\frac{1}{2}\theta\right) + 2$

Key Features:

Amp: 3 period: 4π p.s.: none
v.s. up 2

$f(x) = -3\cos\frac{1}{2}\theta$



20. $f(x) = \csc\left(\frac{1}{4}\theta + \pi\right) - 3$

Key Features:

amp: 1 period: 8π p.s. 4π left
v.s. down 3

$f(x) = \sin\frac{1}{4}\theta$

