

Section 7.5 Solving Rational Equations

Name: Key

Solve a rational equation by cross-multiplying.

1. $\frac{2}{x} = \frac{3}{x-4}$

$2(x-4) = 3x$
 $2x - 8 = 3x$ $x = -8$

2. $\frac{7}{r+9} = \frac{3}{r+3}$

$7(r+3) = 3(r+9)$
 $7r + 21 = 3r + 27$

$\rightarrow 4r = 6$
 $r = \frac{3}{2}$

3. $\frac{6}{k+3} = \frac{5}{k-7}$

$6(k-7) = 5(k+3)$
 $6k - 42 = 5k + 15$
 $k = 57$

4. $\frac{x}{5} = \frac{x+4}{7}$

$7x = 5(x+4)$
 $7x = 5x + 20$
 $2x = 20$ $x = 10$

Solve a rational equation with one solution.

5. $\frac{x-4}{4} + \frac{x}{3} = 6$

LCD: 12

$12\left(\frac{x-4}{4}\right) + 12\left(\frac{x}{3}\right) = 12 \cdot 6$
 $3(x-4) + 4x = 72$
 $3x - 12 + 4x = 72$
 $7x = 84$
 $x = 12$

6. $\frac{k+1}{3} - \frac{k}{5} = 3$

LCD: 15

$15\left(\frac{k+1}{3}\right) - 15\left(\frac{k}{5}\right) = 15 \cdot 3$
 $5(k+1) - 3k = 45$
 $5k + 5 - 3k = 45$
 $2k = 40$
 $k = 20$

7. $\frac{2a-3}{6} = \frac{2a}{3} + \frac{1}{2}$

LCD: 6

$6\left(\frac{2a-3}{6}\right) = 6\left(\frac{2a}{3}\right) + 6\left(\frac{1}{2}\right)$
 $2a - 3 = 4a + 3$
 $2a = 6$
 $a = -3$

8. $\frac{2}{r^2} - \frac{1}{r} = 1$

LCD: r^2

$r^2\left(\frac{2}{r^2}\right) - r^2\left(\frac{1}{r}\right) = r^2 \cdot 1$
 $2 - r = r^2$
 $r^2 + r - 2 = 0$
 $(r+2)(r-1) = 0$
 $r = -2$ or 1

9. $\frac{3}{m^2} = \frac{m-4}{3m^2} + \frac{2}{m^2}$

LCD: $3m^2$

$3m^2\left(\frac{3}{m^2}\right) = 3m^2\left(\frac{m-4}{3m^2}\right) + 3m^2\left(\frac{2}{m^2}\right)$
 $9 = (m-4) + 6$
 $9 = m + 2$
 $m = 7$

10. $\frac{n-7}{4n} + \frac{3}{2} = \frac{6}{n}$

LCD: $4n$

$4n\left(\frac{n-7}{4n}\right) + 4n\left(\frac{3}{2}\right) = 4n\left(\frac{6}{n}\right)$
 $n-7 + 6n = 24$
 $7n = 31$
 $n = \frac{31}{7}$

Solve a rational equation. (Check for extraneous solutions).

11. ~~$\frac{x+1}{x} = 1 - \frac{k^2-3k-4}{4}$~~

12. $\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$

LCD: $(x+1)(x-1)$ or x^2-1

$(4x+1)(x-1) = 12 + 3(x^2-1)$
 $4x^2 - 3x - 1 = 12 + 3x^2 - 3$
 $-3x^2 - 9 = 3x^2 + 9$

$x^2 - 3x - 10 = 0$
 $(x-5)(x+2) = 0$
 $x = 5$ or -2

13. $\frac{4}{k^2-8k+12} = \frac{k}{k-2} + \frac{1}{k-6}$
 LCD: $(k-2)(k-6)$

$4 = k(k-6) + (k-2)$
 $4 = k^2 - 6k + k - 2$
 $0 = k^2 - 5k - 6$
 $0 = (k-6)(k+1)$
 $k = 6$
 $k = -1$

14. $\frac{5}{p-5} - \frac{p^2}{p-5} = 2$
 LCD: $(p-5)$

$5 - p^2 = 2(p-5)$
 $5 - p^2 = 2p - 10$
 $p^2 - 2p - 15 = 0$
 $(p-5)(p+3) = 0$
 $p = 5$
 $p = -3$

15. $\frac{-1}{y-3} = \frac{7y+3}{y^2-8y+15} + \frac{3y}{y-5}$
 LCD: $(y-3)(y-5)$

$-(y-5) = 7y+3 + 3y(y-3)$
 $-y+5 = 7y+3 + 3y^2 - 9y$
 $0 = 3y^2 + y - 2$
 $0 = (3y-2)(y+1)$
 $y = -1$
 $y = 2/3$

16. $\frac{11}{a+2} - \frac{10}{a+5} = \frac{36}{a^2+7a+10}$
 LCD: $(a+2)(a+5)$

$11(a+5) - 10(a+2) = 36$
 $11a + 55 - 10a - 20 = 36$
 $a + 35 = 36$
 $a = 1$

17. $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2-8x+12}$
 LCD: $(x-6)(x-2)$

$1(x-2) + x(x-6) = 4$
 $x-2 + x^2 - 6x = 4$
 $x^2 - 5x - 6 = 0$
 $(x-6)(x+1) = 0$
 $x = 6$
 $x = -1$

18. $\frac{8x^2}{x^2-9} - \frac{4x}{x+3} = \frac{2}{x-3}$
 LCD: $(x+3)(x-3)$

$8x^2 - 4x(x-3) = 2(x+3)$
 $8x^2 - 4x^2 + 12x = 2x + 6$
 $4x^2 + 10x - 6 = 0$
 $2(2x^2 + 5x - 3) = 0$
 $2(2x-1)(x+3) = 0$
 $x = 1/2$
 $x = -3$

19. $\frac{2a-3}{a-3} - 2 = \frac{12}{a+3}$
 LCD: $(a+3)(a-3)$

$(2a-3)(a+3) - 2(a^2-9) = 12(a-3)$
 $2a^2 + 3a - 9 - 2a^2 + 18 = 12a - 36$
 $3a + 9 = 12a - 36$
 $-9a = -45$
 $a = 5$

20. $\frac{y}{y+3} - \frac{3}{y-5} = 2$
 LCD: $(y+3)(y-5)$

$y(y-5) - 3(y+3) = 2(y+3)(y-5)$
 $y^2 - 5y - 3y - 9 = 2(y^2 - 2y - 15)$
 $y^2 - 8y - 9 = 2y^2 - 4y - 30$
 $0 = y^2 + 4y - 21$
 $0 = (y+7)(y-3)$
 $y = -7$
 $y = 3$

21. $y - \frac{6}{y} = 5$
 LCD: y

$y(y) - 6 = 5(y)$
 $y^2 - 5y - 6 = 0$
 $(y-6)(y+1) = 0$
 $y = 6$ or -1

22. $\frac{4u}{u+3} + u = \frac{8}{u+3}$
 LCD: $u+3$

$4u + u(u+3) = 8$
 $4u + u^2 + 3u = 8$
 $u^2 + 7u - 8 = 0$
 $(u+8)(u-1) = 0$
 $u = -8$ or 1