

## 5.1 – Operations on Functions/Composition

I can use function notation when adding, subtracting, multiplying and dividing two functions.

I understand what a composition of functions is & can find  $f(g(x))$  &  $g(f(x))$  when given two sets of ordered pairs.

I can find  $f(g(x))$  and  $g(f(x))$  when given functions  $f(x)$  and  $g(x)$ .

I can find the domain and range of a composite function.

### Operations on Functions:

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\text{Domain: } g(x) \neq 0$$

### Operations of Functions and their Domains

#9  $f(x) = x - 1$  and  $g(x) = 5x - 2$

Find the functions of  $f + g$ ,  $f - g$ ,  $fg$ , and  $\frac{f}{g}$  and their domains.

$$(f+g)(x) = (x-1) + (5x-2) = 6x-3$$

$$(f-g)(x) = (x-1) - (5x-2) = -4x+1$$

$$(fg)(x) = (x-1)(5x-2) = 5x^2 - 7x + 2$$

$$\left(\frac{f}{g}\right)(x) = \frac{x-1}{5x-2} \quad D: x \neq \frac{2}{5}$$

#11  $f(x) = 3x$  and  $g(x) = -2x + 6$

Find the functions of  $f + g$ ,  $f - g$ ,  $fg$ , and  $\frac{f}{g}$  and their domains. (you try)

### Composition of Functions

Given two functions  $f$  and  $g$ , the

**composite function**  $f \circ g$  is defined by:

$$\begin{cases} (f \circ g)(x) = f(g(x)) \\ (g \circ f)(x) = g(f(x)) \end{cases}$$

Example 1: Compose Functions

For each pair of functions, find  $[f \circ g](x)$  and  $[g \circ f](x)$ , if they exist. State the domain and range for each composed function.

$$f = \{(1, 8), (0, 13), (15, 11), (14, 9)\}, g = \{(8, 15), (5, 1), (10, 14), (9, 0)\}$$

To find  $f \circ g$ , evaluate  $g(x)$  first. Then use the range to evaluate  $f(x)$ .

start with domain from  $g(x)$

$$f(g(8)) = f(15) = 11$$

$$f(g(5)) = f(1) = 8$$

$$f(g(10)) = f(14) = 9$$

$$f(g(9)) = f(0) = 13$$

To find  $g \circ f$ , evaluate  $f(x)$  first. Then use the range to evaluate  $g(x)$ .

start with domain of  $f(x)$

$$g(f(1)) = g(8) = 15$$

$$g(f(0)) = g(13) = \text{undefined}$$

$$g(f(15)) = g(11) = \text{undefined}$$

$$g(f(14)) = g(9) = 0$$

For each pair of functions, find  $f \circ g$  and  $g \circ f$ , if they exist. State the domain and range for each composed function.

*You try*

17.  $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$   
 $g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

19.  $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$   
 $g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$

Example 2: Compose Functions:

b.  $f(x) = 2x - 5, g(x) = 4x$

$(f \circ g)(x) = f(g(x))$       Composition of functions

$f(4x) = 2(4x) - 5$   
 $8x - 5$

$(g \circ f)(x) = g(f(x))$

$g(2x - 5) = 4(2x - 5)$   
 $8x - 20$

27.  $f(x) = 2x$   
 $g(x) = x + 5$

$f(g(x)) = f(x + 5)$   
 $= 2(x + 5) = 2x + 10$   
 $g(f(x)) = g(2x) = (2x) + 5$   
 $= 2x + 5$

29.  $f(x) = x + 5$   
 $g(x) = 3x - 7$

*You try*

31.  $f(x) = x^2 + 6x - 2$   
 $g(x) = x - 6$

*You try*

$f(x) = 5x$ , and  $g(x) = -2x + 1$ ,  $h(x) = x^2 + 6x + 8$

Find each value,

*You try*

41.  $f[g(-2)]$

$g(-2) = -2(-2) + 1$   
 $= 5$

42.  $g[h(3)]$

$f(5) = 5(5)$   
 $= 25$