Sketching Graphs of Rational Functions:

1- **Factor.**
Factor the numerator and denominator.

2- **Intercepts.**
Find the x-intercept - set the **numerator** equal to 0 and solve.
Find the y-intercept - set x = 0 and solve.

3- **Vertical Asymptotes.**
Set the **denominator** equal to 0 and solve.
These are always x = # equations.

4- **Horizontal Asymptotes.**
To find the horizontal asymptotes look at the degree of the numerator and the denominator.

If $numerator < denominator$, then asymptote is $y = 0$.

If $numerator = denominator$, then asymptote is
$$y = \frac{leading\ coefficient\ of\ numerator}{leading\ coefficient\ of\ denominator}.$$

If $numerator > denominator$, then *there is* no horizontal asymptote. This creates a slant asymptote.

5- **Slant Asymptotes.**
To find the slant asymptotes divide the numerator by the denominator. Slant asymptotes are always in the form
$$y = mx + b$$

$$if\ r(x) = {}^{P(x)}\!/_{Q(x)}$$

$$r(x) = ax + b + \frac{R(x)}{Q(x)}$$

6- **Sketch the graph.** Graph the information provided by the first five steps. Then plot as many additional points as needed to fill in the rest of the graph of the function.
See $if\ y \to +\infty\ or\ y \to -\infty$ on each side of every vertical asymptote by using test points.

Section 4.5 Graphing of Rational Functions

1-6 Find the x- and y-intercepts of the rational function:

2. $s(x) = \frac{3x}{x-5}$

x-int: (0,0)
y-int (0,0)

4. $r(x) = \frac{2}{x^2+3x-4}$

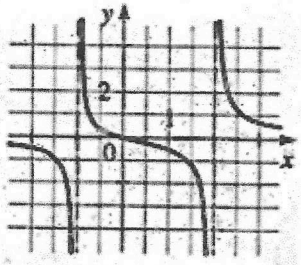
x-int: none
y-int: $-1/2$

3. $r(x) = \frac{x^2-x-2}{x-6} = \frac{(x-2)(x+1)}{x-6}$

x-int: (2,0)(-1,0)
y-int: $(0, -1/3)$

7-10 Identify the x- and y-intercepts and the vertical and horizontal asymptotes

8.

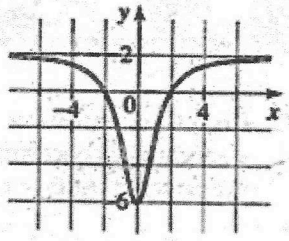


x-int (0,0)
y-int (0,0)

V.A.
$x=-1$
$x=2$

H.A.
$y=0$

10.



x-int:
(-2,0)(2,0)
y-int:
(0,-6)

V.A. none
H.A. $y=2$

11-20 Find the horizontal and vertical asymptotes (if any):

14. $y = \frac{2x-4}{x^2+2x+1} = \frac{2(x-2)}{(x+1)^2}$

V.A. $x=-1$
H.A. $y=0$

20. $y = \frac{x^3+3x^2}{x^2-4} = \frac{x^2(x+3)}{(x+2)(x-2)}$

V.A. $x=\pm 2$
H.A. none

17. $y = \frac{6x-2}{x^2+5x-6}$ $\frac{2(3x-1)}{(x+6)(x-1)}$

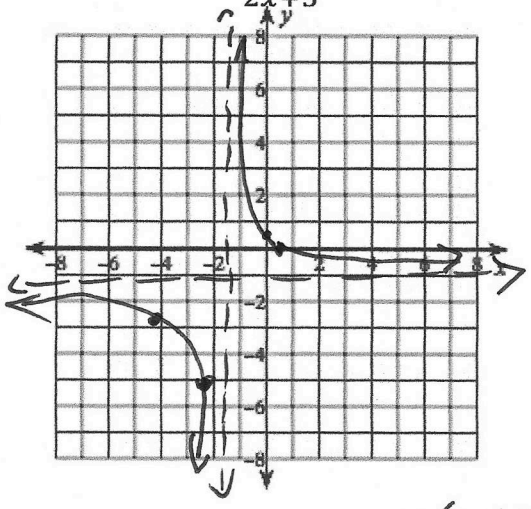
V.A.
$x=-6$
$x=1$

H.A.
$y=0$

Graphing Rational Functions without a calculator:

29- 52 Find the intercepts and asymptotes, and then sketch a graph of the function:

32. $s(x) = \frac{1-2x}{2x+3}$



x-int $(1/2, 0)$

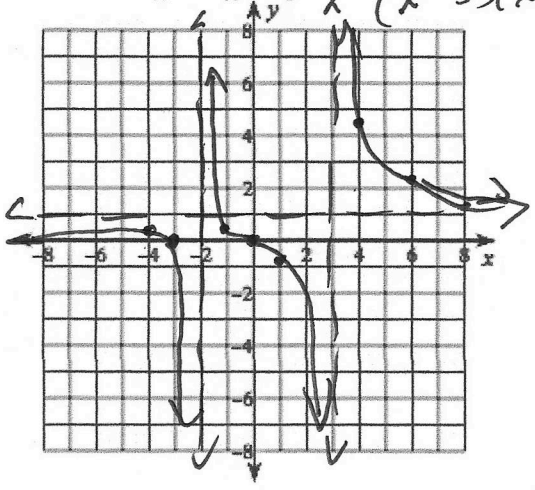
y-int $(0, 1/3)$

V.A. $x = -3/2$

H.A. $y = -1$

x	y
-2	-5
-4	$-9/5$

48. $r(x) = \frac{x^2+3x}{x^2-x-6} = \frac{x(x+3)}{(x-3)(x+2)}$



x-int : $(0,0)$
$(-3,0)$

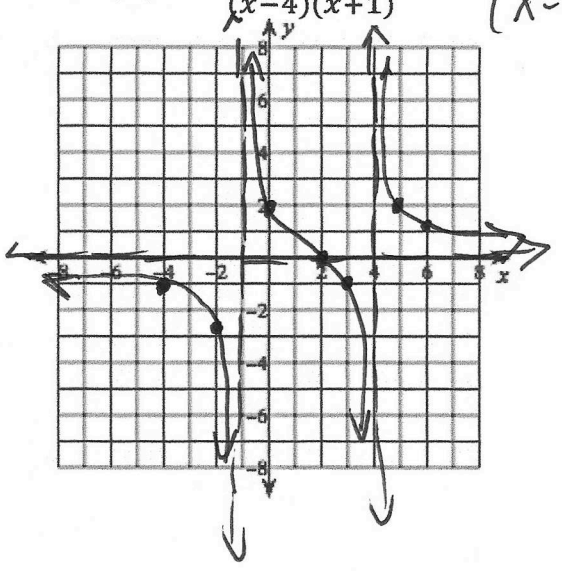
y-int : $(0,0)$

V.A. $x = 3, x = -2$

H.A. $y = 1$

x	y
-5	$5/12$
-1	$1/2$
1	$-2/3$
2	$-5/2$
4	$14/3$
6	$27/12$

35. $r(x) = \frac{4x-8}{(x-4)(x+1)} = \frac{4(x-2)}{(x-4)(x+1)}$



x-int : $(2,0)$

y-int $(2,0)$

V.A. $x = 4$
$x = -1$

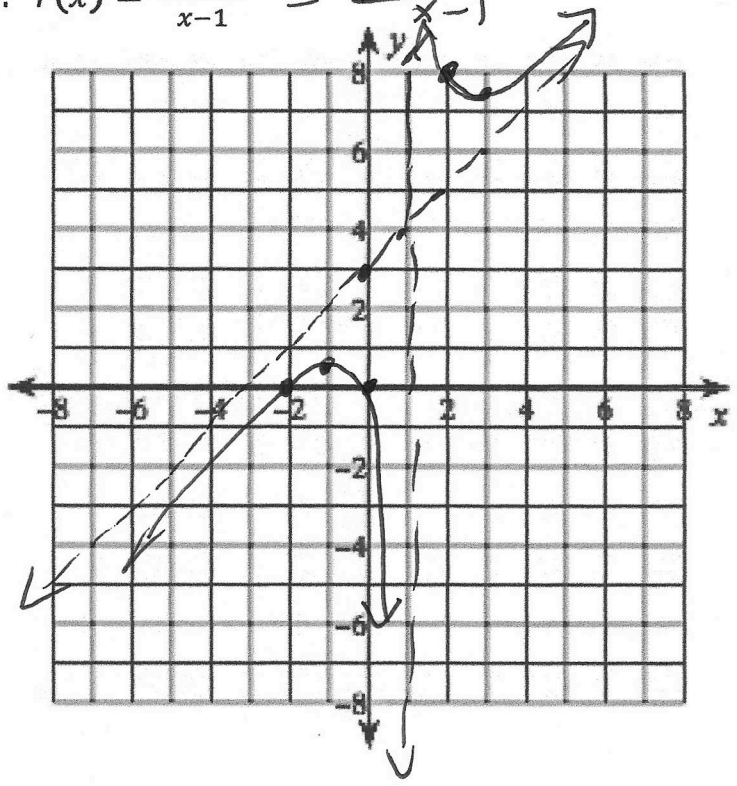
H.A. $y = 0$

x	y
-4	-1
-2	$-8/3$
3	-1
5	2
6	$8/7$

53-60

Find the slant asymptote, the vertical asymptotes, and sketch a graph of the function:

54. $r(x) = \frac{x^2+2x}{x-1} = \frac{x(x+2)}{x-1}$



x-int: (0,0)(-2,0)

y-int: (0,0)

V.A.: $x = 1$

S.A.: $y = x + 3$

$$x-1 \;\overline{)\; x^2+2x}$$

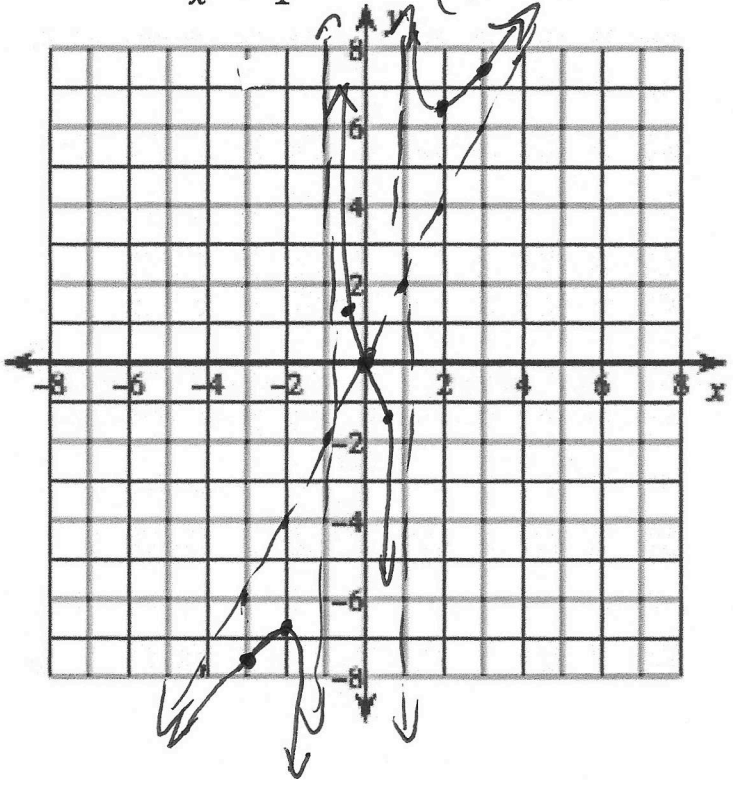
$x + 3$

$x^2 - x$

$3x$

x	y
-1	1/2
2	8
3	15/2

60. $r(x) = \frac{2x^3+2x}{x^2-1} = \frac{2x(x^2+1)}{(x+1)(x-1)}$



x-int: (0,0)

y-int: (0,0)

V.A: $x = \pm 1$

S.A: $y = 2x$

$$x^2-1 \;\overline{)\; 2x^3+2x}$$

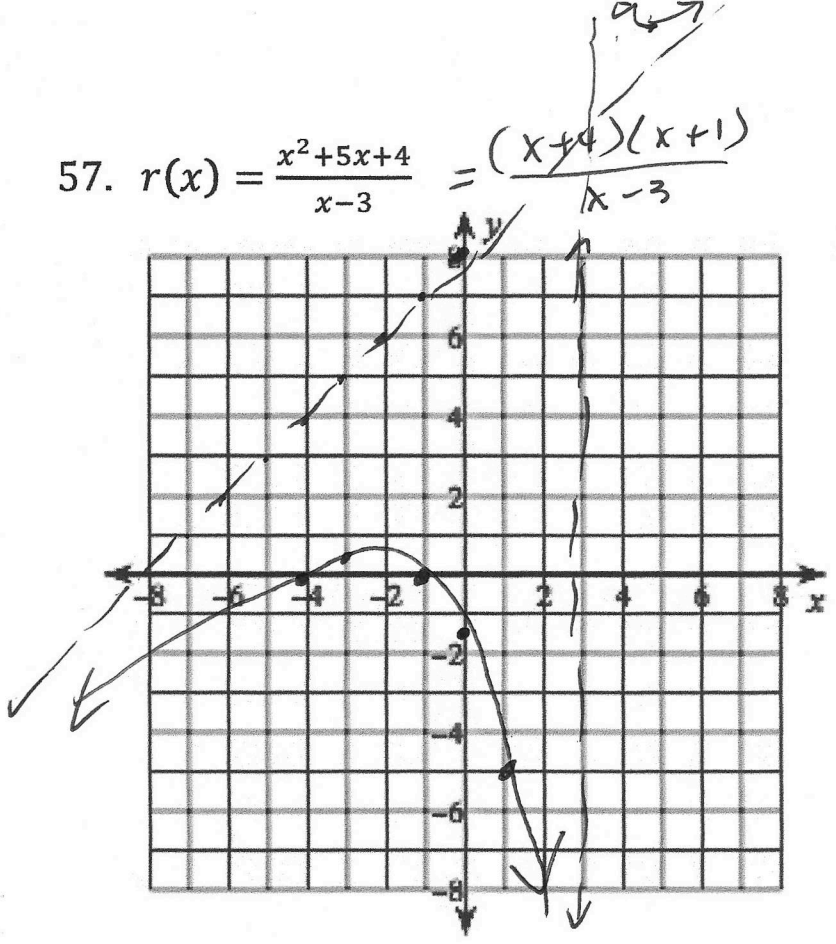
$2x + 0$

$2x^3 - 2x$

$4x$

x	y
-3	-15/2
-2	-20/3
2	20/3
3	15/2
-1/2	5/3
1/2	-5/3

57. $r(x) = \frac{x^2+5x+4}{x-3} = \frac{(x+4)(x+1)}{x-3}$



x-int: $(-4,0)$ $(-1,0)$

y-int: $(0, -4/3)$

V.A. $x = 3$

S.A. $y = x + 8$

$$x-3 \overline{)\, x^2+5x+4} \quad x+8$$

$$x^2 - 3x$$

$$8x + 4$$

x	y
4	40
6	$70/3$
1	-8
-3	$-1/3$

61. $f(x) = \frac{2x^2+6x+6}{x+3}$ $\quad 2\frac{(x^2+3x+3)}{x+3}$

put in desmos and change window

$f(x) = 2x$