Section 4.5 Graphing of Rational Functions

1-6 Find the x- and y-intercepts of the rational function:

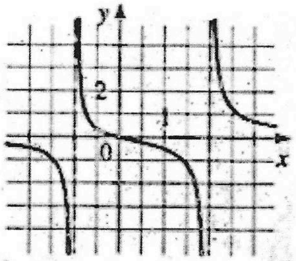
2. $s(x) = \frac{3x}{x-5}$

4. $r(x) = \frac{2}{x^2+3x-4}$

3. $r(x) = \frac{x^2 - x - 2}{x - 6}$

7-10 Identify the x- and y-intercepts and the vertical and horizontal asymptotes

8.



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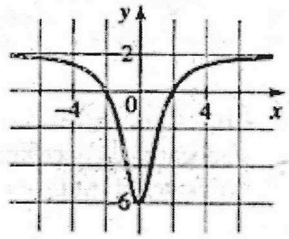


11-20 Find the horizontal and vertical asymptotes (if any):

14. $y = \frac{2x-4}{x^2+2x+1}$

20. $y = \frac{x^3+3x^2}{x^2-4}$

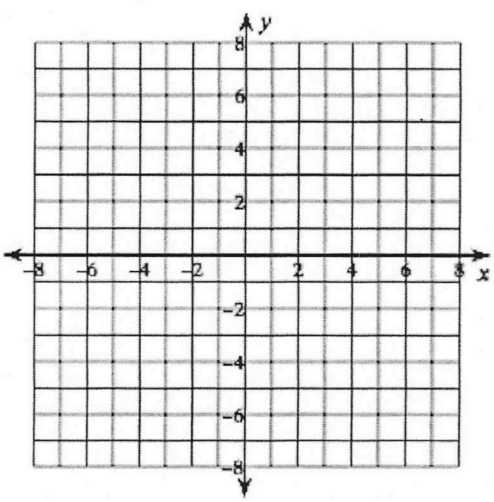
17. $y = \frac{6x-2}{x^2+5x-6}$

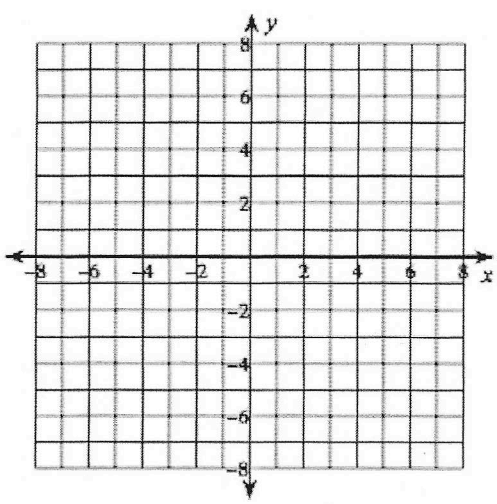
Graphing Rational Functions without a calculator:

29- 52 Find the intercepts and asymptotes, and then sketch a graph of the function:

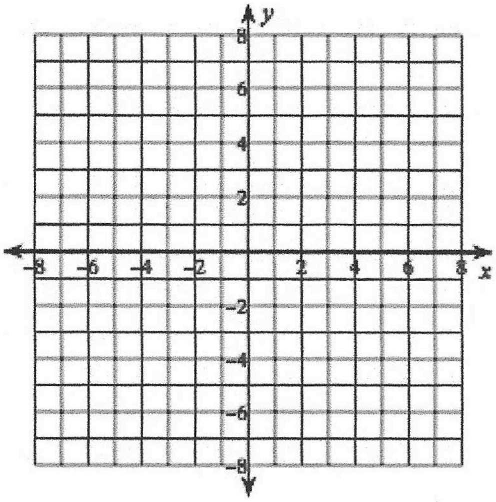
32. $s(x) = \frac{1-2x}{2x+3}$

48. $r(x) = \frac{x^2+3x}{x^2-x-6}$





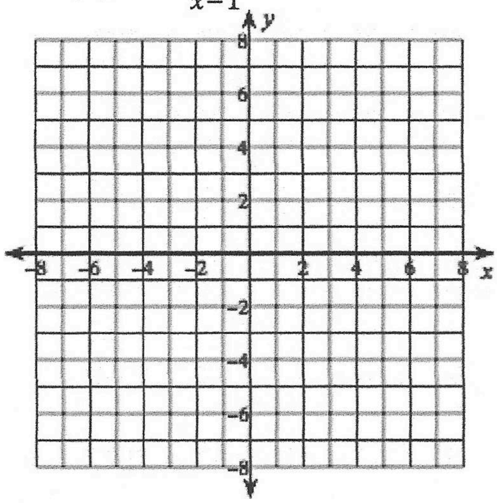
35. $r(x) = \frac{4x-8}{(x-4)(x+1)}$



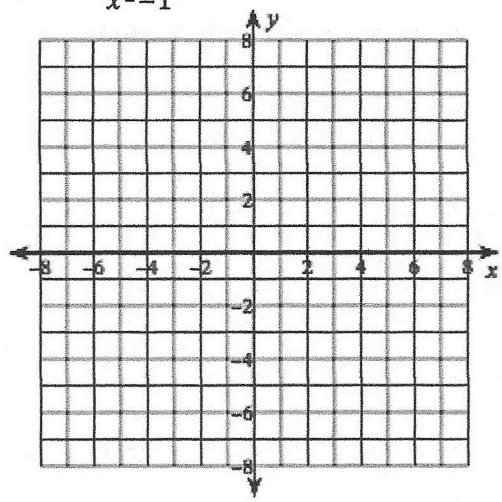
53-60

Find the slant asymptote, the vertical asymptotes, and sketch a graph of the function:

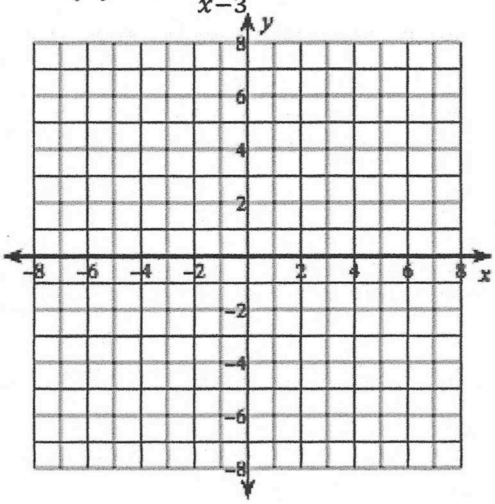
54. $r(x) = \frac{x^2+2x}{x-1}$



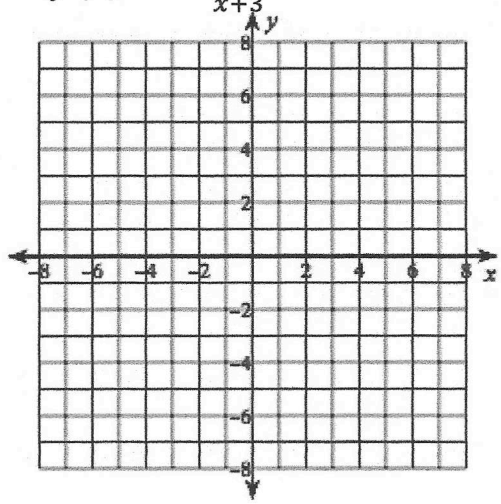
60. $r(x) = \frac{2x^3+2x}{x^2-1}$



57. $r(x) = \frac{x^2+5x+4}{x-3}$



61. $f(x) = \frac{2x^2+6x+6}{x+3}$



Sketching Graphs of Rational Functions:

1- **Factor.**
Factor the numerator and denominator.

2- **Intercepts.**
Find the x-intercept - set the **numerator** equal to 0 and solve.
Find the y-intercept - set x = 0 and solve.

3- **Vertical Asymptotes.**
Set the **denominator** equal to 0 and solve.
These are always x = # equations.

4- **Horizontal Asymptotes.**
To find the horizontal asymptotes look at the degree of the numerator and the denominator.

If $numerator < denominator$, then asymptote is $y = 0$.

If $numerator = denominator$, then asymptote is
$$y = \frac{leading\ coefficient\ of\ numerator}{leading\ coefficient\ of\ denominator}.$$

If $numerator > denominator$, then *there is* no horizontal asymptote. This creates a slant asymptote.

5- **Slant Asymptotes.**
To find the slant asymptotes divide the numerator by the denominator. Slant asymptotes are always in the form
$$y = mx + b$$

$$if\ r(x) = {}^{P(x)}/_{Q(x)}$$

$$r(x) = ax + b + \frac{R(x)}{Q(x)}$$

6- **Sketch the graph.** Graph the information provided by the first five steps. Then plot as many additional points as needed to fill in the rest of the graph of the function.
See $if\ y \rightarrow +\infty\ or\ y \rightarrow -\infty$ on each side of every vertical asymptote by using test points.