Notes Section 4.4

Factoring a Polynomial Completely
Factor a Polynomial with Quadratic Formula
- Use the Conjugate Zeros Theorem
- With Irrational Zeros
- With Complex Zeros

Find a Polynomial with Specified Zeros

1-12 A polynomial P is given.
(a) Find all zeros of P, real and complex.
(b) Factor completely.

8. $P(x) = x^4 + 6x^2 + 9$.

$(x^2+3)^2$; $(x+i\sqrt{3})^2(x-i\sqrt{3})^2$

5. $P(x) = x^4 + 2x^2 + 1$

$(x^2+1)^2$; $(x+i)^2(x-i)^2$

10. $P(x) = x^3 - 8$

$(x-2)(x^2+2x+4)$;

$(x-2)(x-(-1+i\sqrt{3}))(x-(-1-i\sqrt{3}))$

$\frac{-2 \pm \sqrt{4-4(4)}}{2}$

$= \frac{-2 \pm \sqrt{-12}}{2}$

$\frac{-2 \pm 2i\sqrt{3}}{2}$

$= -1 \pm i\sqrt{3}$

13-30 Factor the polynomial completely and find all its zeros.
State the multiplicity of each zero.

20. $P(x) = x^4 - 625$

$(x^2-25)(x^2+25)$

$(x+5)(x-5)(x+5i)(x-5i)$

$\pm 5, \pm 5i$

28. $P(x) = x^5 + 7x^3$

$x^3(x^2+7)$

$x^3(x-i\sqrt{7})(x+i\sqrt{7})$

0 mult 3, $\pm i\sqrt{7}$

23. $P(x) = x^3 + x^2 + 9x + 9$

$x^2(x+1) + 9(x+1)$

$(x+1)(x^2+9)$

$(x+1)(x+3i)(x-3i)$

$-1, \pm 3i$

31-40 Find a polynomial with integer coefficients that satisfies the given conditions.

34. Q has degree 3, and zeros 0 and i.

$x(x-i)(x+i)$

$x(x^2+1) = x^3+x$

36. Q has degree 3, and zeros -3 and $1 + i$.

$(x+3)(x-(1+i))(x-(1-i))$

$(x-1-i)(x-1+i)$ (use $(a+b)(a-b) = a^2-b^2$)

$((x-1)-i)((x-1)+i)$

$(x-1)^2 - (i)^2 = x^2-2x+1-(-1) = x^2-2x+2$

$(x+3)(x^2-2x+2)$

x^3+x^2-4x+6

31. Q has degree 2, and zeros $1+i$ *and* $1-i$.

$(x-(1+i))(x-(1-i))$

$((x-1)-i)((x-1)+i)$

$(x-1)^2-(i)^2 = x^2-2x+1-(-1)$

x^2-2x+2

41-56 Find all zeros of the polynomial.

44. $P(x) = x^3+7x^2+18x+18$

-3 | 1 7 18 18

| -3 -12 -18

1 4 6 0

x^2+4x+6

$\frac{-4\pm\sqrt{16-4(6)}}{2} = \frac{-4\pm\sqrt{-8}}{2}$

$= \frac{-2\pm 2i\sqrt{2}}{2} = -1\pm i\sqrt{2}$

$-3, -1\pm i\sqrt{2}$

53. $P(x) = x^4-6x^3+13x^2-24x+36$

3 | 1 -6 13 -24 36

| 3 -9 12 -36

3 | 1 -3 4 -12 0

| 3 0 12

1 0 4 0

$x^2+4=0$

$x^2=-4$

$x=\pm 2i$

3 mult 2, $\pm 2i$

55. $P(x) = x^5-3x^4+12x^3-28x^2+27x-9$

1 | 1 -3 12 -28 27 -9

| 1 -2 10 -18 9

1 | 1 -2 10 -18 9 0

| 1 -1 9 -9

1 -1 9 -9 0

1 | 1 -1 9 -9

| 1 0 9

1 0 9 0

$x^2+9=0$

$x^2=-9;\ x=\pm 3i$

1 mult 3, $\pm 3i$

57-62 A polynomial P is given.

(a) Factor P into linear and irreducible quadratic factors with real coefficients.

(b) Factor P completely into linear factors with complex coefficients.

60. $P(x) = x^4+8x^2+16$

(a) $(x^2+4)^2$

(b) $(x+2i)^2(x-2i)^2$

61. $P(x) = x^6-64$ (use $a^3-b^3=(a-b)(a^2+ab+b^2)$)

$(x^3-8)(x^3+8)$

(a) $(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

(b) $(x-2)(x+2)(x-(-1+i\sqrt{3}))(x-(-1-i\sqrt{3}))$

$(x-(1+i\sqrt{3}))(x-(1-i\sqrt{3}))$

x^2+2x+4

$\frac{-2\pm\sqrt{4-4(4)}}{2}$

$\frac{-2\pm\sqrt{-12}}{2}$

$\frac{-2\pm 2i\sqrt{3}}{2}$

$= -1\pm i\sqrt{3}$

x^2-2x+4

$\frac{2\pm\sqrt{4-4(4)}}{2}$

$\frac{2\pm\sqrt{-12}}{2}$

$\frac{2\pm 2i\sqrt{3}}{2}$

$1\pm i\sqrt{3}$