Trig Section 3.4
Use the Product-Sum Formulas to find the exact value of an expression
Use the Sum-to-Product Formulas to write trig expressions
Use the Product-Sum Formulas and the Sum-to-Product Formulas to verify identities

Product-Sum Formulas: $\cos(-x) = \cos x$ $\sin(-x) = -\sin x$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)] \qquad \cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)] \qquad \sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

In Exercises 1 to 8, write each expression as the sum or difference of two functions.

2. $2 \sin 4x \sin 2x$

$\frac{1}{2}[2\cos(u-v) - \cos(u+v)]$
$\frac{1}{2}(2\cos 2x - \cos 6x)$
$= \boxed{\cos 2x - \cos 6x}$

4. $\cos 3x \cos 5x$

$\frac{1}{2}[\cos(u+v) + \cos(u-v)]$
$\frac{1}{2}(\cos 8x + \cos(-2x))$
$\boxed{\frac{1}{2}(\cos 8x + \cos 2x)}$

In Exercises 9 to 16, find the exact value of each expression.
Do not use a calculator.

10. $\sin 105° \cos 15°$

$\frac{1}{2}[\sin 120 + \sin 90]$
$\frac{1}{2}\left[\frac{\sqrt{3}}{2} + 1\right] = \boxed{\frac{\sqrt{3}}{4} + \frac{1}{2}}$

13. $\sin \frac{13\pi}{12} \cos \frac{\pi}{12}$

$\frac{1}{2}\left(\sin \frac{7\pi}{6} + \sin \pi\right)$
$\frac{1}{2}\left(-\frac{1}{2} + 0\right)$
$= \boxed{-\frac{1}{4}}$

Sum-to-Product Formulas:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \qquad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \qquad \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

In Exercises 17 to 32, write each expression as a product of two functions.

18. $\cos 5\theta - \cos 3\theta$

$-2 \sin \frac{8\theta}{2} \sin \frac{2\theta}{2}$
$\boxed{-2 \sin 4\theta \sin \theta}$

20. $\sin 7\theta - \sin 3\theta$

$2 \cos \frac{10\theta}{2} \sin \frac{4\theta}{2}$
$\boxed{2 \cos 5\theta \sin 2\theta}$

In Exercises 33 to 48, verify the identity.

34. $2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

$\cos \alpha - \beta - \cos(\alpha + \beta)$

$\cos \alpha \cancel{\cos \beta} + \sin \alpha \sin \beta - (\cos \alpha \cos \beta - \sin \alpha \sin \beta)$

$- \cos \alpha \cancel{\cos \beta} + \sin \alpha \sin \beta$

$= 2 \sin \alpha \sin \beta$

36. $\sin 5x \cos 3x = \sin 4x \cos 4x + \sin x \cos x$

use Product - Sum rule

$\sin 4x \cos 4x + \sin x \cos x$

$= \frac{1}{2}(\sin 8x + \sin 0) + \frac{1}{2}(\sin 2x + \sin 0)$

$= \frac{1}{2}(8x + 2x)$ (use - Sum to product rule)

$\frac{1}{2}\left(2 \sin \frac{10x}{2} \cos \frac{6x}{2}\right)$

$= \sin 5x \cos 3x$

39. $\sin 3x - \sin x = 2 \sin x - 4\, sin^3 x$

use Sum-to-product

$\sin 3x - \sin x = 2 \cos \frac{4x}{2} \sin \frac{2x}{2}$

$2 \cos 2x \sin x$ (use product sum)

$2\left(\frac{1}{2}(\sin 3x - \sin x)\right)$

$= \sin 3x - \sin x$

$- \sin x$

$\sin 3x = \sin(x + 2x)$

$\sin x \cos 2x + \cos x \sin 2x$

$\sin x (1 - 2\sin^2 x) + \cos x (2 \sin x \cos x)$

$2 \sin x \cos^2 x$

$2 \sin x (1 - \sin^2 x)$

$\sin x - 2\sin^3 x + 2 \sin x - 2 \sin^3 x$ $- \sin x$

$\boxed{2 \sin x - 4 \sin^3 x}$