

Trig Section 3.4

Use the Product-Sum Formulas to find the exact value of an expression

Use the Sum-to-Product Formulas to write trig expressions

Use the Product-Sum Formulas and the Sum-to-Product Formulas to verify identities

Product-Sum Formulas:

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$$

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

In Exercises 1 to 8, write each expression as the sum or difference of two functions.

2. $2 \sin 4x \sin 2x$

4. $\cos 3x \cos 5x$

In Exercises 9 to 16, find the exact value of each expression.

Do not use a calculator.

10. $\sin 105^\circ \cos 15^\circ$

13. $\sin \frac{13\pi}{12} \cos \frac{\pi}{12}$

Sum-to-Product Formulas:

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

In Exercises 17 to 32, write each expression as a product of two functions.

18. $\cos 5\theta - \cos 3\theta$

20. $\sin 7\theta - \sin 3\theta$

In Exercises 33 to 48, verify the identity.

34. $2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

36. $\sin 5x \cos 3x = \sin 4x \cos 4x + \sin x \cos x$

39. $\sin 3x - \sin x = 2 \sin x - 4 \sin^3 x$