Trig Section 3.3
Use the Double Angle Identities to find exact values
Use the Half Angle Identities to find exact values
Use the Double Angle and Half Angle formulas to prove identities.

Double Angle Formulas

$\sin(2\theta) = \sin(\theta+\theta)$

$= \sin\theta\cos\theta + \sin\theta\cos\theta$

$= \boxed{2\sin\theta\cos\theta}$

$\cos(\theta+\theta)$

$\cos(2\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$

$\boxed{\cos^2\theta - \sin^2\theta}$

or $= \cos^2\theta - (1-\cos^2\theta)$

$\boxed{2\cos^2\theta - 1}$

or $= \cos^2\theta - \sin^2\theta$

$1-\sin^2\theta - \sin^2\theta$

$\boxed{1-2\sin^2\theta}$

$\tan(2\theta) = \tan(\theta+\theta)$

$= \dfrac{\tan\theta + \tan\theta}{1-\tan\theta\tan\theta}$

$= \boxed{\dfrac{2\tan\theta}{1-\tan^2\theta}}$

In Exercises 1 to 8, write each trigonometric expression in terms of a single trigonometric function.

2. $2\sin 3\theta \cos 3\theta = \sin 2(3\theta) = \sin 6\theta$

$2\sin\theta\cos\theta = \sin 2\theta$

4. $2\cos^2 2\beta - 1 = \cos 2(2\beta) = \boxed{\cos 4\beta}$

$2\cos^2\theta - 1 = \cos 2\theta$

6. $\cos^2 6\alpha - \sin^2 6\alpha = \cos 2(6\alpha)$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

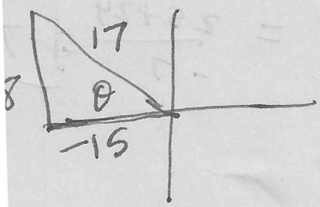
$= \cos 12\alpha$

8. $\dfrac{2\tan 4\theta}{1-\tan^2 4\theta} = \tan 2(4\theta) = \boxed{\tan 8\theta}$

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

In Exercises 25 to 36, find the exact value of $\sin 2\theta$, $\cos 2\theta$, *and* $\tan 2\theta$ given the following information.

27. $\sin\theta = \frac{8}{17}$ *θ is in Quadrant II.*



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{8}{17}\right)\left(\frac{-15}{17}\right)$

$= \boxed{\dfrac{-240}{289}}$

$\cos 2\theta = 1-2\sin^2\theta$

$= 1-2\left(\frac{8}{17}\right)^2$

$= \boxed{\dfrac{161}{189}}$

or $\cos^2\theta - \sin^2\theta$

$\left(-\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$

$\frac{225}{289} - \frac{64}{289} = \boxed{\frac{161}{189}}$

$\tan 2\theta$

$= \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(-\frac{8}{15}\right)}{1-\left(-\frac{8}{15}\right)^2}$

$\left(\dfrac{\frac{-16}{15}}{1-\frac{64}{225}}\right)\dfrac{225}{225}$

$= \dfrac{-240}{161}$

In Exercises 49 to 94, verify the given identity.

$\cos 2x = \cos^2 x - \sin^2 x$

53. $\dfrac{1+\cos 2x}{\sin 2x} = \cot x$

$\sin 2x = 2\sin x\cos x$

$\dfrac{1+(\cos^2 x - \sin^2 x)}{2\sin x\cos x}$

$= \dfrac{1-\sin^2 x + \cos^2 x}{2\sin x\cos x}$

$\dfrac{\cos^2 x + \cos^2 x}{2\sin x\cos x}$

$= \dfrac{2\cos^2 x}{2\sin x\cos x} = \dfrac{\cos x}{\sin x} = \cot x$

56. $\dfrac{\cos^2 x - \sin^2 x}{2\sin x\cos x} = \cot 2x$

$\cos^2 x - \sin^2 x = \cos 2x$

$2\sin x\cos x = \sin 2x$

$\dfrac{\cos 2x}{\sin 2x} = \cot 2x$

Half-Angle Formulas:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$ $\qquad\qquad$ $\cos\frac{x}{2} = \pm\sqrt{\frac{\cos x + 1}{2}}$

The choice of the + or − sign depends on the quadrant in which $\frac{x}{2}$ *lies*

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$ (or) $\frac{1-\cos x}{\sin x}$

In Exercises 9 to 24, use the half-angle identities to find the exact value of each trigonometric expression.

12. tan 165°

$= \tan\frac{330}{2}$

$= \frac{1-\cos 330}{\sin 330^\circ} = \frac{1-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -2+\sqrt{3}$

18. $\cos\frac{5\pi}{8}$

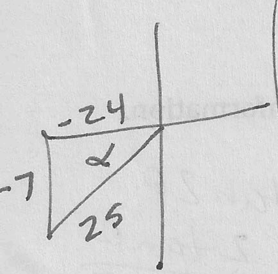
$= \cos\frac{\frac{5\pi}{4}}{2} = -\sqrt{\frac{\cos\frac{5\pi}{4}+1}{2}}$

$= -\sqrt{\frac{-\frac{\sqrt{2}}{2}+1}{2}} = -\sqrt{\frac{-\sqrt{2}+2}{4}}) - \frac{\sqrt{-\sqrt{2}+}}{2}$

In Exercises 37 to 48,

Find the exact values of the sine, cosine, and tangent of $\frac{\alpha}{2}$ given the following information.

38. $\sin\alpha = -\frac{7}{25}$ $\quad\alpha$ *is in Quadrant III*.



$\sin\frac{\alpha}{2} = +\sqrt{\frac{1+\cos x}{2}}$

$= +\sqrt{\frac{1+24/25}{2}} = \sqrt{\frac{49}{50}}$

$= \frac{\sqrt{7}}{5\sqrt{2}} = \frac{7\sqrt{2}}{10}$

$\cos\frac{\alpha}{2} = -\sqrt{\frac{\frac{-24}{25}+1}{2}}$

$= -\sqrt{\frac{-24+25}{50}}$

$= -\sqrt{\frac{1}{50}}$

$= -\frac{\sqrt{1}}{5\sqrt{2}} = -\frac{\sqrt{2}}{10}$

$\tan\frac{\alpha}{2}$

$= \frac{1-\frac{-24}{25}}{\frac{-7}{25}}$

$= \frac{25+24}{-7} = [-$

In Exercises 49 to 94, verify the given identity.

(Grouping)

63. $\cos^2 x - 2\sin^2 x\cos^2 x - \sin^2 x + 2\sin^4 x = \cos^2 2x$

$\cos^2 x(1-2\sin^2 x) - \sin^2 x(1-2\sin^2 x)$

$(\cos^2 x - \sin^2 x)(1-2\sin^2 x)$

$\cos 2x\,(\cos 2x) = \boxed{\cos^2 2x}$

75. $2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$

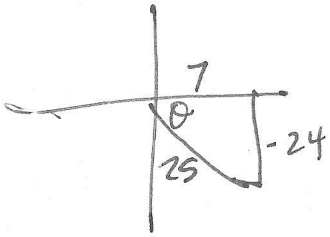
$2\sin\theta\cos\theta = \sin 2\theta$

$2\sin\frac{x}{2}\cos\frac{x}{2} = \sin 2\left(\frac{x}{2}\right)$

$= \sin x$

In Exercises 25 to 36, find the exact value of $\sin 2\theta$, $\cos 2\theta$, *and* $\tan 2\theta$ given the following information.

29. $\tan\theta = -\frac{24}{7}$ *θ is in Quadrant IV.*



$\sin 2\theta$	$\cos 2\theta$	$\tan 2\theta$
$2\sin\theta\cos\theta$	$\cos^2\theta-\sin^2\theta$	$\frac{2\tan\theta}{1-\tan^2\theta}$
$2\left(\frac{-24}{25}\right)\left(\frac{7}{25}\right)$	$\left(\frac{7}{25}\right)^2-\left(\frac{-24}{25}\right)^2$	$\frac{2\left(-\frac{24}{7}\right)}{1-\left(-\frac{24}{7}\right)^2} = \frac{-\frac{48}{7}}{1-\frac{576}{49}}$
$\frac{-336}{625}$	$\frac{49}{625}-\frac{576}{625}$	$\frac{-336}{-527}$
	$\boxed{-527/625}$	

In Exercises 9 to 24, use the half-angle identities to find the exact value of each trigonometric expression.

17. $\sin\frac{7\pi}{8}$

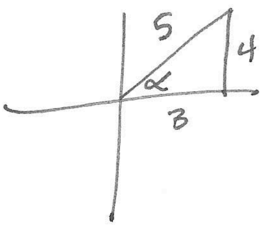
$$\sin\frac{\frac{7\pi}{4}}{2} = +\sqrt{\frac{1-+\frac{\sqrt{2}}{2}}{2}}$$

$$= +\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$\boxed{= \frac{\sqrt{2-\sqrt{2}}}{2}}$$

In Exercises 37 to 48,
Find the exact values of the sine, cosine, and tangent of $\frac{\alpha}{2}$ given the following information.

41. $\tan\alpha = \frac{4}{3}$ *α is in Quadrant I.*



$\sin\frac{\alpha}{2}$

$$= +\sqrt{\frac{1-3/5}{2}}$$

$$\sqrt{\frac{5-3}{10}} = \frac{\sqrt{2}}{\sqrt{10}} \quad \frac{\sqrt{5}}{5}$$

$\cos\frac{\alpha}{2}$

$$+\sqrt{\frac{\frac{3}{5}+1}{2}} = \frac{3+5}{10} = \sqrt{\frac{4}{5}}$$

$$\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\frac{\alpha}{2} = \frac{1-\frac{3}{5}}{4/5} = \frac{5-3}{4} = \frac{1}{2}$$

In Exercises 49 to 94, verify the given identity.

76. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos 2\left(\frac{x}{2}\right)$$

$$= \cos x$$

In Exercises [illegible] to [illegible] find the exact values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ given the following information.

[illegible]

In Exercises [illegible] to [illegible] [illegible]

[illegible]

In Exercises 47 to 52,
Find the exact values of the sine, cosine, and tangent of $\frac{\theta}{2}$ given the following information.

51. $\tan\theta = [illegible]$, θ is in Quadrant I.

[illegible]

In Exercises [illegible] to 94, verify the given equation.

76. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$

[illegible]