

Trigonometry Section 2.3 Trigonometric Functions of Any Angle:

Find the value of the six trigonometric functions given a point.

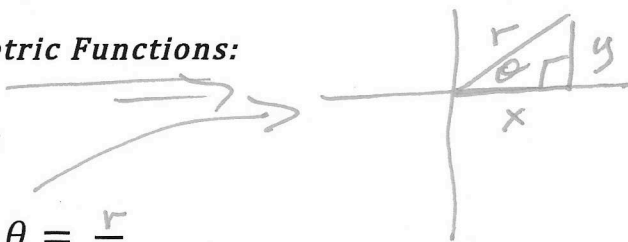
Signs of the trigonometric functions depending on the quadrant

Find a reference angle given any angle between $0^\circ \leq \theta \leq 360^\circ$ or $0 \leq \theta \leq 2\pi$

General Definitions of Trigonometric Functions:

$$x^2 + y^2 = r^2$$

The Six Trigonometric Functions:



$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

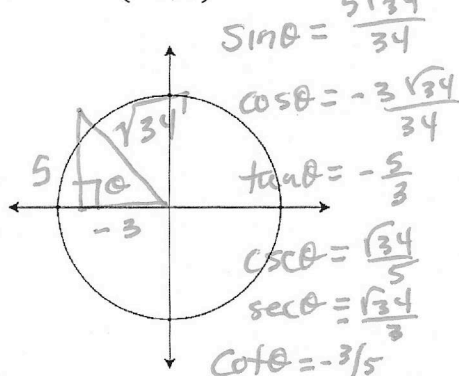
$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \quad x \neq 0$$

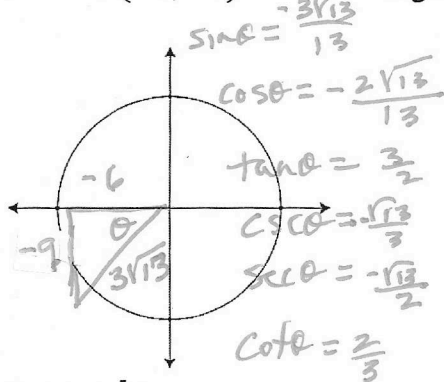
$$\cot \theta = \frac{x}{y} \quad y \neq 0$$

In Exercises 1 to 8, find the value of each of the six trigonometric functions for the angle whose terminal side passes through the given point.

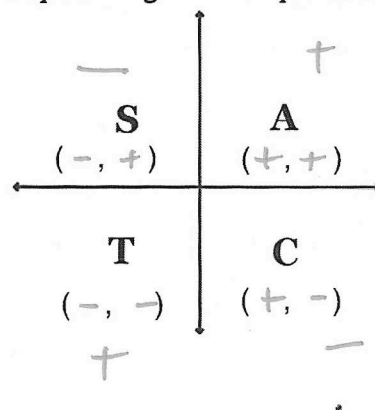
4- $P(-3, 5)$



6- $P(-6, -9)$



Signs of the trigonometric functions depending on the quadrant



Find a reference angle given any angle

between $0^\circ \leq \theta \leq 360^\circ$ or $0 \leq \theta \leq 2\pi$

A reference angle is:

- acute
- positive
- measured from the terminal side of the x-axis

To find the **reference angle** for any angle θ between 0 and 360° or 0 and 2π .

Quadrant I: θ is reference angle

Quadrant III: $\theta - 180^\circ$ or $\theta - \pi$

Quadrant II:

$$180 - \theta$$

$$\text{or } \pi - \theta$$

Quadrant IV:

$$360 - \theta \quad \text{or} \quad 2\pi - \theta$$

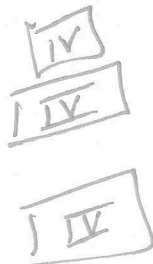
In Exercises 9 to 14, let θ be an angle in standard position.

State the quadrant in which the terminal side of θ lies

10. $\tan \theta < 0, \sin \theta < 0$

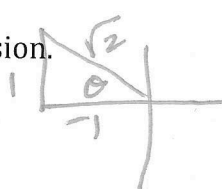
12. $\sin \theta < 0, \cos \theta > 0$

14. $\tan \theta < 0, \cos \theta < 0$

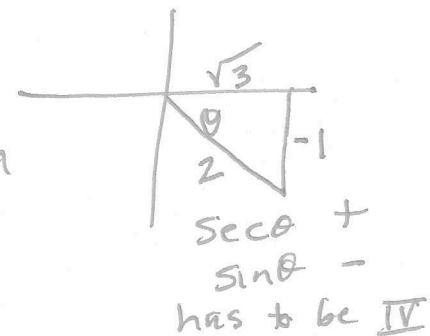


In Exercises 15 to 24, find the value of each expression.

16. $\cot \theta = -1, 90^\circ < \theta < 180^\circ$; find $\cos \theta$.



24. $\sec \theta = \frac{2\sqrt{3}}{3}$ and $\sin \theta = -\frac{1}{2}$; find $\cot \theta$.



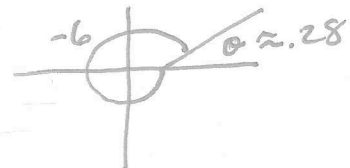
In Exercises 25 to 36, find the measure of the reference angle θ' for the given angle θ .

28. $\theta = 48^\circ$

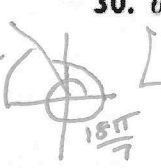
48°

30. $\theta = -6$

.28

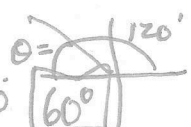


32. $\theta = \frac{18}{7}\pi$



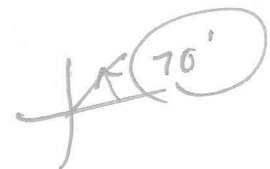
34. $\theta = 840^\circ$

$840 - 720 = 120$



36. $\theta = -650^\circ$

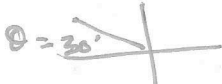
$\frac{+720}{70}$



In Exercises 37 to 48, use the Reference Angle Theorem to find the exact value of each trigonometric function.

40. $\sec 150^\circ$

$\sec 30^\circ = -\frac{2\sqrt{3}}{3}$



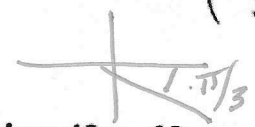
42. $\cot \frac{7}{6}\pi$

$\cot \frac{\pi}{6} = \sqrt{3}$



44. $\tan \left(-\frac{\pi}{3}\right)$

$\tan \frac{\pi}{3} = -\sqrt{3}$



48. $\cos 570^\circ$

$570 - 360 = 210$

$\cos 30^\circ = -\frac{\sqrt{3}}{2}$



In Exercises 49 to 60, use a calculator to approximate the given trigonometric functions to six significant digits.

52. $\cot 398^\circ = \frac{1}{\tan 398^\circ} \approx 1.2799$

or $\frac{1}{\tan 38^\circ}$

54. $\sec 740^\circ$

$\frac{1}{\cos 740^\circ} \approx 1.0642$

or $\frac{1}{\cos 20^\circ}$

56. $\cos \frac{3\pi}{7} \approx .2225$