6.3 – Properties of Logarithms

I can use properties of logarithms to condense logarithms.
I can use properties of logarithms to expand logarithms.
I can use the properties of logarithms to solve equations

PROPERTIES OF LOGARITHMS

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m$ ____ $\log_b$ ____

Quotient Property $\log_b \frac{m}{n} = \log_b m$ ____ $\log_b$ ____

Power Property $\log_b m^n =$ ________

Your Notes

When you are expanding or condensing an expression involving logarithms, you may assume any variables are positive.

Example 2 ***Expand a logarithmic expression***

Expand $\log_3 \frac{7x^2}{y}$.

$\log_3 \frac{7x^2}{y} =$ ________________ **Quotient property**

$=$ ____________________ **Product property**

$=$ ____________________ **Power property**

Example 3 ***Condense a logarithmic expression***

Condense $\log 2 + 3 \log 3 - \log 9$.

$\log 2 + 3 \log 3 - \log 9$

$= \log 2 +$ ________ $- \log 9$ **Power property**

$= \log$ ________ $- \log 9$ **Product property**

$= \log \frac{\quad}{\quad}$ **Quotient property**

$=$ ______ **Simplify.**

Solve Logarithmic Equations

Solve the logarithmic equation.

$\log_3 x + \log_3 (x-4) = \log_3 12$

Use a property of logarithms to combine the left side of the equation.

Use the equality property of logarithms to write and solve a new equation.

Check for extraneous solutions in the original equation.

Example 6 ***Check for extraneous solutions***

Solve $\log 5x + \log(x - 1) = 2$.

$\log 5x + \log(x - 1) = 2$		**Write original equation.**
$\log[\_\_\_\_\_\_\_\_\_\_] = 2$		**Product property of logarithms**
__________	= _____	**Exponentiate each side.**
__________	= _____	$b^{\log_b x} = x$
__________	= _____	**Distributive property**
__________	= ___	**Write in standard form.**
__________	= ___	**Divide each side by ___.**
__________	= ___	**Factor.**
______ or ______		**Zero product property**

Solve each equation. Check your solutions.

1 $\log_{10} 27 = 3 \log_{10} x$

2 $\log_4 5 + \log_4 x = \log_4 60$

3 $\log_5 y - \log_5 8 = \log_5 1$

6.3 – Properties of Logarithms

I can use properties of logarithms to condense logarithms.
I can use properties of logarithms to expand logarithms.
I can use the properties of logarithms to solve equations

PROPERTIES OF LOGARITHMS

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Your Notes

When you are expanding or condensing an expression involving logarithms, you may assume any variables are positive.

Example 2 ***Expand a logarithmic expression***

Expand $\log_3 \frac{7x^2}{y}$.

$\log_3 \frac{7x^2}{y} = \log_3 7x^2 - \log_3 y$ **Quotient property**

$= \log_3 7 + \log_3 x^2 - \log_3 y$ **Product property**

$= \log_3 7 + 2\log_3 x - \log_3 y$ **Power property**

Example 3 ***Condense a logarithmic expression***

Condense log 2 + 3 log 3 − log 9.

$\log 2 + 3 \log 3 - \log 9$

$= \log 2 + \log 3^3 - \log 9$ **Power property**

$= \log 2(3)^3 - \log 9$ **Product property**

$= \log \frac{2(3)^3}{9}$ **Quotient property**

$= \log 6$ **Simplify.**

Solve Logarithmic Equations

Solve the logarithmic equation.

$\log_3 x + \log_3 (x-4) = \log_3 12$

$\log_3 x(x-4) = \log_3 12$ — Use a property of logarithms to combine the left side of the equation.

$x(x-4) = 12$ — Use the equality property of logarithms to write and solve a new equation.

$x^2 - 4x = 12$
$x^2 - 4x - 12 = 0$
$(x-6)(x+2) = 0$ — Check for extraneous solutions in the original equation.

$x = 6$ or -2 $x = -2$ is extraneous

Solution $x = 6$

Example 6 ***Check for extraneous solutions***

Solve $\log 5x + \log(x - 1) = 2$.

$\log 5x + \log(x - 1) = 2$	**Write original equation.**
$\log[\underline{5x(x-1)}] = 2$	**Product property of logarithms**
$\underline{10^{\log 5x(x-1)}} = \underline{10^2}$	**Exponentiate each side.**
$\underline{5x(x-1)} = \underline{100}$	**$b^{\log_b x} = x$**
$\underline{5x^2 - 5x} = \underline{100}$	**Distributive property**
$\underline{5x^2 - 5x - 100} = \underline{0}$	**Write in standard form.**
$\underline{x^2 - x - 20} = \underline{0}$	**Divide each side by $\underline{5}$.**
$\underline{(x-5)(x+4)} = \underline{0}$	**Factor.**
$\underline{x = 5}$ or $\underline{\cancel{x = -4}}$	**Zero product property**

extraneous

Solve each equation. Check your solutions.

1 $\log_{10} 27 = 3 \log_{10} x$

$\log_{10} 27 = \log_{10} x^3$

$27 = x^3$

$\boxed{x = 3}$

2 $\log_4 5 + \log_4 x = \log_4 60$

$\log_4 5x = \log_4 60$

$5x = 60$

$x = 12$

3 $\log_5 y - \log_5 8 = \log_5 1$

$\log_5 \frac{y}{8} = \log_5 1$

$\frac{y}{8} = 1$

$\boxed{y = 8}$