Notes Section 4.3

Find the rational zeros of a polynomial

Using the rational zeros theorem

Using synthetic division

Using the quadratic formula

THE RATIONAL ZERO THEOREM

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has ______ coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } \square}{\text{factor of leading coefficient } \square}$$

1-6 List all possible rational zeros given by the Rational Zeros Theorem (But don't check to see which actually are zeros).

2- $Q(x) = x^4 - 3x^3 - 6x + 8$

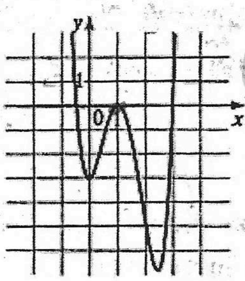
$\frac{1 \cdot 2\ 4\ 8}{1}$ $\pm(1, 2, 4, 8)$

4- $P(x) = 6x^4 - x^2 + 2x + 12$

$\frac{1, 2, 3, 4, 6, 12}{1, 2, 3, 6}$

$\pm\left(1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right)$

9. $P(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$



1	2	-9	9	1	-3
		2	-7	2	3
3	2	-7	2	3	0
		6	-3	-3	
	2	-1	-1	0	

$2x^2 - x - 1$

$(2x+1)(x-1)$

$x = -\frac{1}{2}, 1$

11-38 Find all the rational zeros of the polynomial:

12- $P(x) = x^3 - 7x^2 + 14x - 8$

1	1	-7	14	-8
		1	-6	8
	1	-6	8	0

$x^2 - 6x + 8$

$(x-2)(x-4)$

Zeros: 1, 2, 4

22- $P(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

1	1	-2	-3	8	-4
		1	-1	-4	4
	1	-1	-4	4	0

2	1	-1	-4	4
		2	2	-4
	1	1	-2	0

$x^2 + x - 2$

$(x+2)(x-1)$

Zeros: $(1 \text{ (mult 2)}, \pm 2)$

24- $P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

2	6	-7	-12	3	2
		12	10	-4	-2
	6	5	-2	-1	0

-1	6	5	-2	-1
		-6	1	1
	6	-1	-1	0

$6x^2 - x - 1$

$(2x-1)(3x+1)$

zeros: $\frac{1}{2}, -\frac{1}{3}, 2, -1$

You try ☺

23- $P(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$

$$\begin{array}{r|rrrrr} -1 & 1 & 7 & 14 & 8 \\ & & -1 & -6 & -8 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 1 & 6 & 7 & -6 & -8 \\ & & 1 & 7 & 14 & 8 \\ \hline & 1 & 7 & 14 & 8 & 0 \end{array}$$

$x^2 + 6x + 8$

$(x+2)(x+4)$

Zeros: $\pm 1, -2, -4$

39-48 Find all the zeros of the polynomial. Use the quadratic formula if necessary.

$\pm\left(1, 2, \frac{1}{3}, \frac{2}{3}\right)$

40. $P(x) = x^3 - 5x^2 + 2x + 12$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 2 & 12 \\ & & 3 & -6 & -12 \\ \hline & 1 & -2 & -4 & 0 \end{array}$$

$\frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)} = \frac{2 \pm \sqrt{20}}{2}$

$\frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

46. $P(x) = 3x^3 - 5x^2 - 8x - 2$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -5 & -8 & -2 \\ & & -1 & 2 & 2 \\ \hline & 3 & -6 & -6 & 0 \end{array}$$

$\frac{6 \pm \sqrt{36 - 4(3)(-6)}}{2(3)}$

$\frac{6 \pm \sqrt{108}}{6}$

$\frac{6 \pm 6\sqrt{3}}{6} = 1 \pm \sqrt{3}$

DESCARTES' RULE OF SIGNS

Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$ be a polynomial function with real coefficients.

- The number of positive real zeros of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.
- The number of negative real zeros of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

57-62 Use Descartes' Rule of Signs to determine how many positive and negative real zeros the polynomial can have. Then determine the total number of zeros.

58. $P(x) = 2x^3 - x^2 + 4x - 7$

$P(x) = + \; - \; + \; -$ 3 or 1 pos

$P(-x) = - \; - \; - \; -$ 0 neg

60. $P(x) = x^4 + x^3 + x^2 + x + 12$

$+ \; + \; + \; + \; +$ no pos real zeros

$+ \; - \; + \; - \; +$ 4 or 2 or 0 ~~po~~ neg real zeros

The Upper and Lower Bounds Theorem: $a \le c \le b$

1. Divide P(x) by x-b (with b ≥ 0) using synthetic division
If the last row has no negative numbers
then b is an upper bound

2.
Divide p(x) by x−a (a ≤ 0)
if the last row has alternating non-positive
& non-negative then a is a
lower bound

63-66 Show that the given values for a and b are lower and upper bounds for the real zeros of the polynomial.

64. $P(x) = 2x^3 + 5x^2 + x - 2;\ a = -3, b = 5$

$$\begin{array}{r|rrrr} 5 & 2 & 5 & 1 & -2 \\ & & 10 & 75 & 380 \\ \hline & 2 & 15 & 76 & 378 \end{array}$$

all positive so 5 is an upper bound

$$\begin{array}{r|rrrr} -3 & 2 & 5 & 1 & -2 \\ & & -6 & 3 & -12 \\ \hline & 2 & -1 & 4 & -14 \end{array}$$

alternating
So −3 is lower bound

66. $P(x) = 3x^4 - 17x^3 + 24x^2 - 9x + 1;\ a = 0, b = 6$

$$\begin{array}{r|rrrrr} 6 & 3 & -17 & 24 & -9 & 1 \\ & & 18 & 6 & 180 & 1026 \\ \hline & 3 & 1 & 30 & 171 & 1027 \end{array}$$

yes upper bound

$$\begin{array}{r|rrrrr} 0 & 3 & -17 & 24 & -9 & 1 \\ & & 0 & 0 & 0 & 0 \\ \hline & 3 & -17 & 24 & -9 & 1 \end{array}$$

alt. yes
0 is lower bound

67-70 Find integers that are upper and lower bounds for the real zeros of the polynomial.

68. $P(x) = 2x^3 - 3x^2 - 8x + 12$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -8 & 12 \\ & & 4 & 2 & -12 \\ \hline & 2 & 1 & -6 & 0 \end{array}$$

no neg entries

$$\begin{array}{r|rrrr} -3 & -2 & -3 & -8 & 12 \\ & & 6 & -9 & 51 \\ \hline & -2 & 3 & -17 & 63 \end{array}$$

alternating
So −3 is lower bound

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -8 & 12 \\ & & 6 & 9 & 3 \\ \hline & 2 & 3 & 1 & 15 \end{array}$$

all (+) so 3 is upper bound.

71-76 Find all rational zeros of the polynomial, and then find the irrational zeros, if any. Whenever appropriate, use the Rational Zeros Theorem, the Upper and Lower Bounds Theorem, Descartes' Rule of Signs, the quadratic formula, or other factoring techniques.

72. $P(x) = 2x^4 + 15x^3 + 31x^2 + 20x + 4$

all positive no changes in sign
so (0 positive zeros)

$$\begin{array}{r|rrrrr} -2 & 2 & 15 & 31 & 20 & 4 \\ & & -4 & -22 & -18 & -4 \\ \hline -\frac{1}{2} & 2 & 11 & 9 & 2 & 0 \\ & & -1 & -5 & -2 & \\ \hline & 2 & 10 & 4 & 0 & \end{array}$$

$2x^2 + 10x + 4$

$2(x^2 + 5x + 2) = 0$

$\dfrac{-5 \pm \sqrt{25 - 4(1)(2)}}{2}$

$\dfrac{-5 \pm \sqrt{17}}{2}$