Trig Section 3.2
Use the Sum and Difference Identities to find exact values
Use the Cofunction Identities to write equivalent expressions
Use the Sum and Difference and Cofunction formulas to prove identities.

Sum and Difference Formulas:

$\sin(x+y) =$

$\sin(x-y) =$

$\cos(x+y) =$

$\cos(x-y) =$

$\tan(x+y) =$

$\tan(x-y) =$

3. $\cos(45° - 30°)$

8. $\sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

11. $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

14. $\sin 167° \cos 107° - \cos 167° \sin 107°$

18. $\dfrac{\tan\frac{\pi}{6} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{3}}$

Cofunction Formulas

$\boldsymbol{\cos\theta} =$

$\boldsymbol{\sin\theta} =$

$\boldsymbol{\sec\theta} =$

$\boldsymbol{\csc\theta} =$

$\boldsymbol{\tan\theta} =$

$\boldsymbol{\cot\theta} =$

In Exercises 19 to 24, use a cofunction identity to write an equivalent expression for the given value.

20. $\cos 80°$

22. $\cot 2°$

24. $\csc 84°$

In Exercises 25 to 36, write each expression in terms of a single trigonometric function.

26. $\sin x \cos 3x + \cos x \sin 3x$

28. $\cos 4x \cos 2x - \sin 4x \sin 2x$

33. $\sin\frac{x}{3}\cos\frac{2x}{3} + \cos\frac{x}{3}\sin\frac{2x}{3}$

In Exercises 37 to 48, find the exact value of the given functions.

39. Given $\sin\alpha = \frac{3}{5}$, α *in Quadrant I, and* $\cos\beta = -\frac{5}{13}$, β *in Quadrant II, find*

a. $\sin(\alpha - \beta)$ b. $\cos(\alpha + \beta)$ c. $\tan(\alpha - \beta)$

43. Given $\cos\alpha = \frac{15}{17}$, α *in Quadrant I, and* $\sin\beta = -\frac{3}{5}$, β *in Quadrant II, find*

a. $\sin(\alpha + \beta)$ b. $\cos(\alpha - \beta)$ c. $\tan(\alpha - \beta)$

In Exercises 49 to 72, verify the identity.

51. $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

53. $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\tan\theta + 1}{1 - \tan\theta}$

62. $\cos 5x \cos 3x + \sin 5x \, \sin 3x = cos^2 x - sin^2 x$

67. $\frac{\cos(\alpha - \beta)}{\sin(\alpha + \beta)} = \frac{\cot\alpha + \tan\beta}{1 + \cot\alpha \tan\beta}$

72. $\cos 3x = 4\, cos^3 x - 3\cos x$

Trig Section 3.2
Use the Sum and Difference Identities to find exact values
Use the Cofunction Identities to write equivalent expressions
Use the Sum and Difference and Cofunction formulas to prove identities.

Sum and Difference Formulas:

$\sin(x+y) =$ sin x cos y + cos x sin y

$\sin(x-y) =$ sin x cos y − cos x sin y

$\cos(x+y) =$ cos x cos y − sin x sin y

$\cos(x-y) =$ cos x cos y + sin x sin y

$\tan(x+y) =$ tan x + tan y / 1 − tan x tan y

$\tan(x-y) =$ tan x + tan y / 1 − tan x tan y

3. $\cos(45° - 30°)$ $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

cos 45 cos 30 + sin 45 sin 30

$\frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2} \quad + \quad \frac{\sqrt{2}}{2} \; \frac{1}{2}$

8. $\sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

11. $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

14. $\sin 167° \cos 107° - \cos 167° \sin 107°$

sin(167 − 107) = sin 90
= 1

18. $\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{3}}$

$\tan\left(\frac{\pi}{6} + \frac{\pi}{3}\right)$

$\tan\frac{\pi}{2}$ und

Cofunction Formulas

cos θ = $\sin\left(\frac{\pi}{2} - \theta\right)$

sin θ = $\cos\left(\frac{\pi}{2} - \theta\right)$

sec θ = $\csc\left(\frac{\pi}{2} - \theta\right)$

csc θ = $\sec\left(\frac{\pi}{2} - \theta\right)$

tan θ = $\cot\left(\frac{\pi}{2} - \theta\right)$

cot θ = $\tan\left(\frac{\pi}{2} - \theta\right.$

In Exercises 19 to 24, use a cofunction identity to write an equivalent expression for the given value.

20. cos 80° sin 10
~~cos 80~~

22. cot 2°
tan 88

24. csc 84°
sec 6

In Exercises 25 to 36, write each expression in terms of a single trigonometric function.

26. $\sin x \cos 3x + \cos x \sin 3x$

sin(4x)

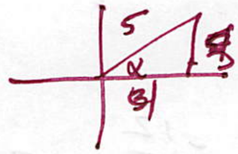
28. $\cos 4x \cos 2x - \sin 4x \sin 2x$

cos 6x

33. $\sin\frac{x}{3}\cos\frac{2x}{3} + \cos\frac{x}{3}\sin\frac{2x}{3}$

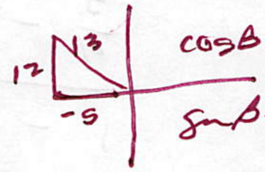
$\sin\left(\frac{x}{3} + \frac{2x}{3}\right) = \sin\frac{3x}{3} = \sin x$

In Exercises 37 to 48, find the exact value of the given functions.



$\tan\alpha = 3/4$
$\sin\alpha = \frac{3}{5}$
$\cos\alpha = \frac{4}{5}$

39. Given $\sin\alpha = \frac{3}{5}$, α *in Quadrant I, and* $\cos\beta = -\frac{5}{13}$, β *in Quadrant II, find*



$\cos\beta = \frac{-5}{13}$
$\sin\beta = \frac{12}{13}$
$\tan\beta = \frac{-12}{5}$

a. $\sin(\alpha - \beta)$ b. $\cos(\alpha + \beta)$ c. $\tan(\alpha - \beta)$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\frac{3}{5}\ \frac{-5}{13} \qquad \frac{4}{5}\ \frac{12}{13}$
$-\frac{15}{65} - \frac{48}{65} = -\frac{63}{65}$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\frac{4}{5}\ \frac{-5}{13} \qquad \frac{3}{5}\ \frac{12}{13}$
$\frac{-20}{65} - \frac{36}{65} = \frac{-56}{65}$

$\frac{\tan\alpha - \tan\beta}{1 - \tan\alpha\tan\beta}$
$\frac{\frac{3}{4} - \frac{-12}{5}}{1 - \frac{3}{4}\left(\frac{-12}{5}\right)} = \frac{63/20}{14/5}$ $\cancel{\frac{63}{20} \cdot \frac{5}{14}}$
$\frac{63}{56}$

43. Given $\cos\alpha = \frac{15}{17}$, α *in Quadrant I, and* $\sin\beta = -\frac{3}{5}$, β *in Quadrant II, find*

a. $\sin(\alpha + \beta)$ b. $\cos(\alpha - \beta)$ c. $\tan(\alpha - \beta)$

$\Rightarrow$

In Exercises 49 to 72, verify the identity.

51. $\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2}$
$\sin\theta(0) + \cos\theta(1)$
$= \cos\theta$

53. $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\tan\theta + 1}{1 - \tan\theta}$

$\frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\left(\tan\frac{\pi}{4}\right)} = \frac{\tan\theta + 1}{1 - \tan\theta(1)}$
$= \frac{\tan\theta + 1}{1 - \tan\theta}$

62. $\cos 5x \cos 3x + \sin 5x\ \sin 3x = cos^2x - sin^2x$

$\cos(5x - 3x)$
$\cos(2x)$
$= \cos(x + x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$

67. $\frac{\cos(\alpha - \beta)}{\sin(\alpha + \beta)} = \frac{\cot\alpha + \tan\beta}{1 + \cot\alpha\tan\beta}$

$\frac{(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}{(\sin\alpha\cos\beta + \sin\alpha\sin\beta)} = \cancel{\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta + \sin\alpha\sin\beta}} + \cancel{\frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta + \sin\alpha\sin\beta}}$

72. $\cos 3x = 4\,cos^3x - 3\cos x$

$\cos 2x\cos x - \sin 2x\sin x$
$\cos x(\cos(x + x)) - \sin(x)(\sin(x + x))$
$\cos x(\cos x\cos x - \sin x\sin x) - \sin x(\sin x\cos x + \sin x\cos x)$
$\cos x(\cos^2 x - \sin^2 x) \qquad \sin x(2\sin x\cos x)$
$2\sin^2 x\cos x$