Trigonometry Section 3.1 Proving Trigonometric Identities

Tangent and Cotangent Identities:

Reciprocal Identities:

Pythagorean Identities:

Guidelines for Proving Trigonometric Identities:

1-

2-

3-

4-

5-

6-

Verify that the equation is not an identity by finding an x-value for which the left side of the equation is not equal to the right side.

2. $tan2x = 2tanx$

6. $\sqrt{1 + tan^2x} = secx$

Use Algebraic Identities:

Verify each identity:

12. $sinx\ cotx\ secx = 1$

16. $(tanx)(1 - cotx) = tanx - 1$

26. $\frac{sinx}{1-cosx} - \frac{sinx}{1+cosx} = 2cotx$

34. $\frac{2sinx\ cotx+sinx-4cotx-2}{2cotx+1} = sinx - 2$

38. $sin^2x - cos^2x = 2sin^2x - 1$

44. $\frac{\frac{1}{sinx}+\frac{1}{cosx}}{\frac{1}{sinx}-\frac{1}{cosx}} = \frac{cos^2x-sin^2x}{1-2cosx\ sinx}$

52. $\frac{1}{1-sinx} - \frac{1}{1+sinx} = 2tanx\ secx$

Trigonometry Section 3.1 Proving Trigonometric Identities

Tangent and Cotangent Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities:

$\csc\theta = \frac{1}{\sin\theta}$ $\sin\theta = 1/\csc\theta$

$\sec\theta = 1/\cos\theta$ $\cos\theta = 1/\sec\theta$

$\cot\theta = 1/\tan\theta$ $\tan\theta = 1/\cot\theta$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
$\sin^2\theta = 1 - \cos^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$1 = \csc^2\theta - \cot^2\theta$
$\cos^2\theta = 1 - \sin^2\theta$	$1 = \sec^2\theta - \tan^2\theta$	$\cot^2\theta = \csc^2\theta - 1$

Guidelines for Proving Trigonometric Identities:

1- Work on complicated side first

2- Trig substitutions using identities

3- Algebraic operations

4- Change everything to Sine ¢ Cosine

5- Work toward's opposite side

6- PRACTICE!!!

Verify that the equation is not an identity by finding an x-value for which the left side of the equation is not equal to the right side.

2. $\tan 2x = 2\tan x$ $x = \frac{\pi}{4}$

$\tan 2\left(\frac{\pi}{4}\right) = 2\tan\left(\frac{\pi}{4}\right)$

$\tan\frac{\pi}{2}$

und. $= 2\left(\frac{\sqrt{2}}{2}\right)$

6. $\sqrt{1 + \tan^2 x} = \sec x$ $x = \pi$

$\sqrt{1 + (\tan\pi)^2} = \sec\pi$

$\sqrt{1 + 0^2}$

$1 \neq -1$

Use Algebraic Identities:

$(a+b)(a-b) = a^2 - b^2$

$(a+b)(a+b) = a^2 + 2ab + b^2$

$(a-b)(a-b) = a^2 - 2ab + b^2$

Verify each identity:

12. $\sin x \cot x \sec x = 1$

$\cancel{\sin x} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \frac{1}{\cancel{\cos x}} = 1$

16. $(tanx)(1 - cotx) = tanx - 1$

$\tan x - \tan x \cot x$

$\tan x - 1$

26. $\frac{sinx}{1-cosx} - \frac{sinx}{1+cosx} = 2cotx$

$$\frac{\sin x(1+\cos x)}{1-\cos^2 x} - \frac{\sin x(1-\cos x)}{1-\cos^2 x} = \frac{\cancel{\sin x} + \sin x\cos x - \cancel{\sin x} + \sin x \cos x}{1-\cos^2 x}$$

$$= \frac{2\sin x\cos x}{\sin^2 x} = \frac{2\cos x}{\sin x} = 2\cot x$$

34. $\frac{2sinx\ cotx + sinx - 4cotx - 2}{2cotx + 1} = sinx - 2$

(grouping)

$$\frac{\sin x(2\cot x + 1) - 2(2\cot x + 1)}{(2\cot x + 1)} = \sin x - 2$$

38. $sin^2x - cos^2x = 2sin^2x - 1$

$\sin^2 x - (1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x = 2\sin^2 x - 1$

44. $\frac{\frac{1}{sinx} + \frac{1}{cosx}}{\frac{1}{sinx} - \frac{1}{cosx}} = \frac{cos^2x - sin^2x}{1 - 2cosx\ sinx}$

$$\frac{\frac{1}{\sin x} + \frac{1}{\cos x}}{\frac{1}{\sin x} - \frac{1}{\cos x}} = \frac{\frac{\cos x}{\sin x\cos x} + \frac{\sin x}{\sin x\cos x}}{\frac{\cos x}{\sin x\cos x} - \frac{\sin x}{\sin x \cos x}} = \frac{\frac{\cos x + \sin x}{\sin x\cos x}}{\frac{\cos x - \sin x}{\sin x\cos x}}$$

$$= \frac{\cos x + \sin x}{\cancel{\sin x\cos x}} \cdot \frac{\cancel{\sin x\cos x}}{\cos x - \sin x}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x} \cdot \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x - 2\sin x\cos x + \sin^2 x}$$

$$\boxed{= \frac{\cos^2 x - \sin^2 x}{1 - 2\sin x\cos x}}$$

52. $\frac{1}{1-sinx} - \frac{1}{1+sinx} = 2tanx\ secx$

$$\frac{(1+\sin x)}{1-\sin^2 x} - \frac{(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{1 + \sin x - 1 + \sin x}{1 - \sin^2 x} = \frac{2\sin x}{\cos^2 x} = 2\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$$

$$= \boxed{2\tan x \sec x}$$