

Section 4.7A

Date _____ Period _____

A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

1) $-4, -2 + \sqrt{10}, 2 + \sqrt{5}$

2) $4, -3, -2, \sqrt{2}$

3) $2, -3, 2\sqrt{2}$

4) $3 + \sqrt{2}$ mult. 2

Solve each equation with the quadratic formula.

5) $x^2 - 6x + 7 = 0$

6) $3x^2 + 4x - 2 = 0$

7) $x^2 - 4x - 2 = 0$

8) $3x^2 + 2x - 4 = 0$

9) $3x^2 - 10x + 6 = 0$

10) $4x^2 - 8x - 1 = 0$

State the possible rational zeros for each function. Then find all zeros.

11) $f(x) = x^3 - 7x^2 - x + 7$

12) $f(x) = x^3 + 5x^2 + 7x + 3$

13) $f(x) = x^3 + 4x^2 - 3x - 12$

14) $f(x) = x^3 + 4x^2 - x - 4$

15) $f(x) = x^3 - 13x^2 + 23x - 11$

16) $f(x) = x^4 - 7x^2 + 12$

17) $f(x) = 3x^3 - x^2 - 3x + 1$

18) $f(x) = 3x^3 - 5x^2 - 11x - 3$

$$19) x^4 + 2x^3 - 14x^2 + 6x + 5 = 0$$

$$20) x^4 - 3x^2 - 4x^2 + 8x + 8 = 0$$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

$$21) 1, \sqrt{6}$$

$$22) 5, 3, 0$$

$$23) -3, 2\sqrt{2}$$

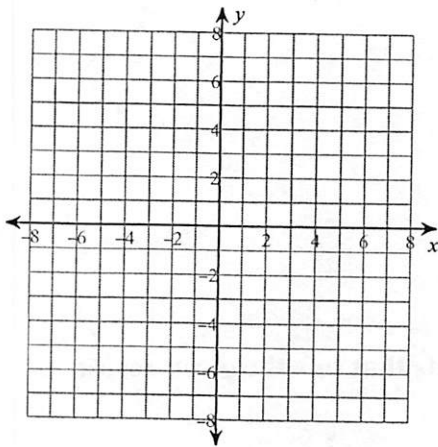
$$24) -3, 2, -5$$

$$25) \sqrt{2}, \sqrt{3}, 2$$

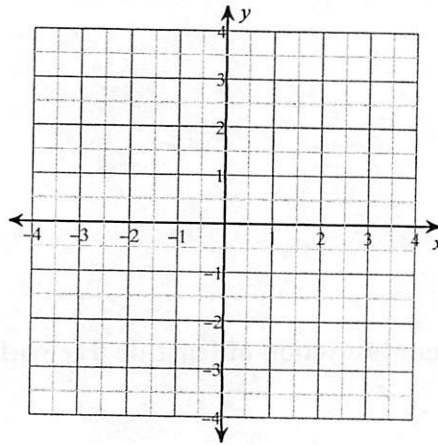
$$26) \sqrt{5}, 3, -2$$

Sketch the graph of each function using its zeros, intercepts, and end behavior.

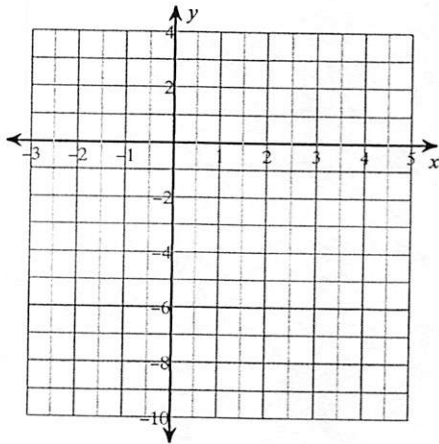
27) $f(x) = x^3 - 2x^2 - 5x + 6$



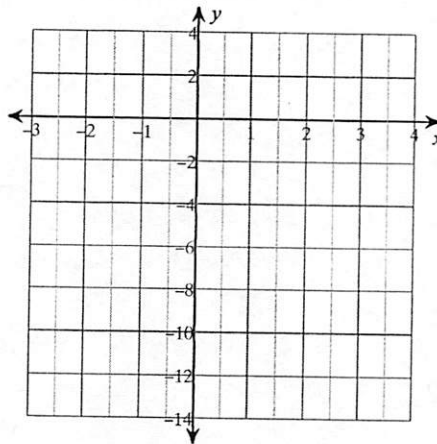
28) $f(x) = 4x^3 + 2x^2 - 4x - 2$



29) $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$



30) $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$



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A polynomial function with rational coefficients has the follow zeros. Find all additional zeros.

1) $-4, -2 + \sqrt{10}, 2 + \sqrt{5}$

$$(-2 - \sqrt{10}); (2 - \sqrt{5})$$

2) $4, -3, -2, \sqrt{2}$

$$-\sqrt{2}$$

3) $2, -3, 2\sqrt{2}$

$$-2\sqrt{2}$$

4) $3 + \sqrt{2}$ mult. 2

$$(3 - \sqrt{2}) \text{ mult. } 2$$

Solve each equation with the quadratic formula.

5) $x^2 - 6x + 7 = 0$

$$\frac{6 \pm \sqrt{36 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{8}}{2}$$
$$= \frac{6 \pm 2\sqrt{2}}{2} = \boxed{3 \pm \sqrt{2}}$$

6) $3x^2 + 4x - 2 = 0$

$$\frac{-4 \pm \sqrt{16 - 4(3)(-2)}}{2(3)} = \frac{-4 \pm \sqrt{40}}{6}$$
$$= \frac{-4 \pm 2\sqrt{10}}{6} = \boxed{\frac{-2 \pm \sqrt{10}}{3}}$$

7) $x^2 - 4x - 2 = 0$

$$\frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2(1)} = \frac{4 \pm \sqrt{24}}{2}$$
$$= \frac{4 \pm 2\sqrt{6}}{2} = \boxed{2 \pm \sqrt{6}}$$

8) $3x^2 + 2x - 4 = 0$

$$\frac{-2 \pm \sqrt{4 - 4(3)(-4)}}{2(3)} = \frac{-2 \pm \sqrt{52}}{6}$$
$$= \frac{-2 \pm 2\sqrt{13}}{6} = \boxed{\frac{-1 \pm \sqrt{13}}{3}}$$

9) $3x^2 - 10x + 6 = 0$

$$\frac{10 \pm \sqrt{100 - 4(3)(6)}}{2(3)} = \frac{10 \pm \sqrt{28}}{6}$$
$$= \frac{10 \pm 2\sqrt{7}}{6} = \boxed{\frac{5 \pm \sqrt{7}}{3}}$$

10) $4x^2 - 8x - 1 = 0$

$$\frac{8 \pm \sqrt{64 - 4(4)(-1)}}{2(4)} = \frac{8 \pm \sqrt{80}}{8}$$
$$\frac{8 \pm 4\sqrt{5}}{8} = \boxed{\frac{2 \pm \sqrt{5}}{2}}$$

State the possible rational zeros for each function. Then find all zeros.

11) $f(x) = x^3 - 7x^2 - x + 7$

Poss: $\pm 1, \pm 7$ Zeros: $\{-1, 1, 7\}$

$$\begin{array}{r} 1 \mid 1 \quad -7 \quad -1 \quad 7 \\ \quad 1 \quad -6 \quad -7 \\ \quad 1 \quad -6 \quad -7 \quad 0 \\ x^2 - 6x - 7 \\ (x-7)(x+1) \end{array}$$

12) $f(x) = x^3 + 5x^2 + 7x + 3$

Poss: $\pm 1, \pm 3$

Zeros: -1 mult 2
 -3

$$\begin{array}{r} -1 \mid 1 \quad 5 \quad 7 \quad 3 \\ \quad -1 \quad -4 \quad -3 \\ \quad 1 \quad 4 \quad 3 \quad 0 \\ x^2 + 4x + 3 \\ (x+3)(x+1) \end{array}$$

13) $f(x) = x^3 + 4x^2 - 3x - 12$

Poss: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r} -4 \mid 1 \quad 4 \quad -3 \quad -12 \\ \quad -4 \quad 0 \quad 12 \\ \quad 1 \quad 0 \quad -3 \quad 0 \\ x^2 - 3 \end{array}$$

Zeros: $\{-4, \pm\sqrt{3}\}$

14) $f(x) = x^3 + 4x^2 - x - 4$

Poss: $\pm 1, \pm 2, \pm 4$

Zeros: $-1, 1, -4$

$$\begin{array}{r} 1 \mid 1 \quad 4 \quad -1 \quad -4 \\ \quad 1 \quad 5 \quad 4 \\ \quad 1 \quad 5 \quad 4 \quad 0 \\ x^2 + 5x + 4 \\ (x+4)(x+1) \end{array}$$

15) $f(x) = x^3 - 13x^2 + 23x - 11$

Poss: $\pm 1, \pm 11$

Zeros: 1 mult 2
 11

$$\begin{array}{r} 1 \mid 1 \quad -13 \quad 23 \quad -11 \\ \quad 1 \quad -12 \quad 11 \\ \quad 1 \quad -12 \quad 11 \quad 0 \\ x^2 - 12x + 11 \\ (x-11)(x-1) \end{array}$$

16) $f(x) = x^4 - 7x^2 + 12$

Poss: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Zeros: ± 2
 $\pm\sqrt{3}$

$$\begin{array}{r} 2 \mid 1 \quad 0 \quad -7 \quad 0 \quad 12 \\ \quad 2 \quad 4 \quad -6 \quad -12 \\ -2 \mid 1 \quad 2 \quad -3 \quad -6 \quad 0 \\ \quad -2 \quad 0 \quad 6 \\ \quad 1 \quad 0 \quad -3 \quad 0 \\ x^2 - 3 \end{array}$$

17) $f(x) = 3x^3 - x^2 - 3x + 1$

Poss: $\pm 1, \pm \frac{1}{3}$

Zeros: $-1, 1, \frac{1}{3}$

$$\begin{array}{r} 1 \mid 3 \quad -1 \quad -3 \quad 1 \\ \quad 3 \quad 2 \quad -1 \\ \quad 3 \quad 2 \quad -1 \quad 0 \\ 3x^2 + 2x - 1 \\ (3x-1)(x+1) \end{array}$$

18) $f(x) = 3x^3 - 5x^2 - 11x - 3$

Poss: $\pm 1, \pm 3, \pm \frac{1}{3}$

Zeros: -1
 $-\frac{1}{3}, 3$

$$\begin{array}{r} -1 \mid 3 \quad -5 \quad -11 \quad -3 \\ \quad -3 \quad 8 \quad 3 \\ \quad 3 \quad -8 \quad -3 \quad 0 \\ 3x^2 - 8x - 3 \\ (3x+1)(x-3) \end{array}$$

$$19) x^4 + 2x^3 - 14x^2 + 6x + 5 = 0$$

Poss: $\pm 1, \pm 5$ Zeros: $1, -5, 1 \pm \sqrt{2}$

$$\begin{array}{r} \underline{1} \quad 1 \quad 2 \quad -14 \quad 6 \quad 5 \\ \quad \quad 1 \quad 3 \quad -11 \quad -5 \\ -5 \quad 1 \quad 3 \quad -11 \quad -5 \quad 0 \\ \quad \quad -5 \quad 10 \quad 5 \\ 1 \quad -2 \quad -1 \quad 0 \end{array}$$

$$\boxed{x^2 - 2x - 1} \rightarrow \frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$20) x^4 - 3x^2 - 4x^2 + 8x + 8 = 0$$

Poss: $\pm 1, \pm 2, \pm 4, \pm 8$
Zeros: $-1, 2, 1 \pm \sqrt{5}$

$$\begin{array}{r} -1 \quad 1 \quad -3 \quad -4 \quad 8 \quad 8 \\ \quad \quad -1 \quad 4 \quad 0 \quad -8 \\ 2 \quad 1 \quad -4 \quad 0 \quad 8 \quad 0 \\ \quad \quad 2 \quad -4 \quad -8 \\ 1 \quad -2 \quad -4 \quad 0 \end{array}$$

$$x^2 - 2x - 4$$

$$\frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2(1)} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

21) $1, \sqrt{6}$

$$(x-1)(x+\sqrt{6})(x-\sqrt{6})$$

$$\boxed{f(x) = x^3 - x^2 - 6x + 6}$$

22) $5, 3, 0$

$$(x-5)(x-3)x$$

$$\boxed{f(x) = x^3 - 8x^2 + 15x}$$

23) $-3, 2\sqrt{2}$

$$(x+3)(x-2\sqrt{2})(x+2\sqrt{2})$$

$$\boxed{f(x) = x^3 + 3x^2 - 8x - 24}$$

24) $-3, 2, -5$

$$(x+3)(x-2)(x+5)$$

$$\boxed{f(x) = x^3 + 6x^2 - x - 30}$$

25) $\sqrt{2}, \sqrt{3}, 2$

$$(x-\sqrt{2})(x+\sqrt{2})(x-\sqrt{3})(x+\sqrt{3})(x-2)$$

$$(x^2 - 2)(x^2 - 3)(x - 2)$$

$$(x^4 - 5x^2 + 6)(x - 2)$$

$$\boxed{f(x) = x^5 - 2x^4 - 5x^3 + 10x^2 + 6x - 12}$$

26) $\sqrt{5}, 3, -2$

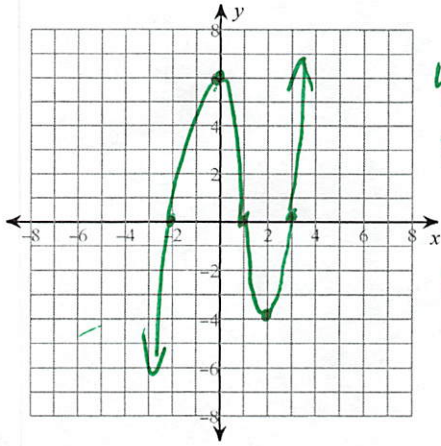
$$(x+\sqrt{5})(x-\sqrt{5})(x-3)(x+2)$$

$$\boxed{f(x) = x^4 - x^3 - 11x^2 + 5x + 30}$$

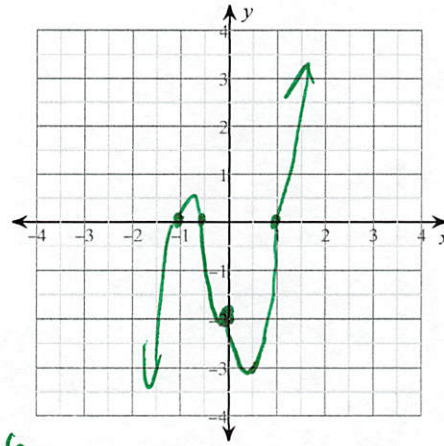
Sketch the graph of each function using its zeros, intercepts, and end behavior.

27) $f(x) = x^3 - 2x^2 - 5x + 6$

28) $f(x) = 4x^3 + 2x^2 - 4x - 2$



y-int (0, 6)
zeros: 1, 3, -2



y-int (0, -2)
zeros: 1, -1, -1/2

(1/2, -3)

$$\begin{array}{r} 1 \quad -2 \quad -5 \quad 6 \\ \underline{1 \quad -1 \quad -6 \quad 0} \\ x^2 - x - 6 \\ (x-3)(x+2) \end{array}$$

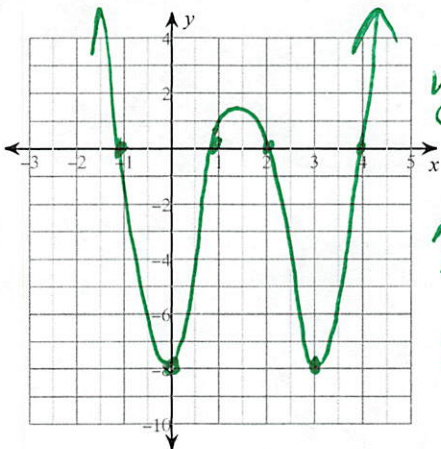
$$\begin{array}{r} 1 \quad -2 \quad -5 \quad 6 \\ \underline{2 \quad 0 \quad -10} \\ 1 \quad 0 \quad -5 \quad -4 \\ (2, -4) \end{array}$$

$$\begin{array}{r} 1 \quad 4 \quad 2 \quad -4 \quad -2 \\ \underline{4 \quad 4 \quad 6 \quad 2} \\ 4 \quad 6 \quad 2 \quad 0 \\ 4x^2 + 6x + 2 = 2(2x^2 + 3x + 1) \\ 2(2x+1)(x+1) \end{array}$$

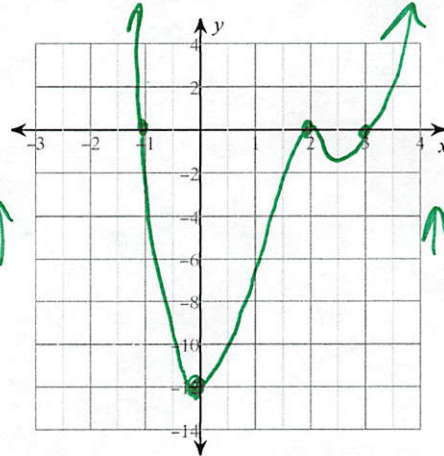
$$\begin{array}{r} 0.5 \quad 4 \quad 2 \quad -4 \quad -2 \\ \underline{4 \quad 2 \quad -2 \quad -1} \\ 4 \quad 2 \quad -2 \quad -3 \end{array}$$

29) $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

30) $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$



y-int (0, -8)
zeros: ±1, 2, 4
(3, -8)



y-int (0, -12)
zeros: -1
2 mult 2
3
(2.5, -0.44)

$$\begin{array}{r} 1 \quad -6 \quad 7 \quad 6 \quad -8 \\ \underline{1 \quad -5 \quad 2 \quad 8 \quad 8} \\ 1 \quad -5 \quad 2 \quad 8 \quad 0 \\ \underline{1 \quad -1 \quad 6 \quad -8} \\ 1 \quad -6 \quad 8 \quad 0 \\ x^2 - 6x + 8 \\ (x-4)(x-2) \end{array}$$

$$\begin{array}{r} 1 \quad -6 \quad 9 \quad 4 \quad -12 \\ \underline{-1 \quad 7 \quad -16 \quad 12} \\ 1 \quad -7 \quad 16 \quad -12 \quad 0 \\ \underline{1 \quad 2 \quad -10 \quad 12} \\ 1 \quad -5 \quad 6 \quad 0 \\ x^2 - 5x + 6 \\ (x-2)(x-3) \end{array}$$

$$\begin{array}{r} 1 \quad -6 \quad 7 \quad 6 \quad -8 \\ \underline{3 \quad -9 \quad -6 \quad 0} \\ 1 \quad -3 \quad -2 \quad 0 \quad -8 \end{array}$$

$$\begin{array}{r} 1 \quad -6 \quad 9 \quad 4 \quad -12 \\ \underline{2.5 \quad -8.75 \quad .625 \quad 11.5625} \\ 1 \quad -3.5 \quad .25 \quad 4.625 \quad -0.4375 \end{array}$$