Review Unit 2 Math 1030 Name: _____

Section 3B Putting Numbers in Perspective

For Problems 1-2 Make an order of magnitude estimates of the following quantities. Explain the assumptions you use in your estimates.

1- Could a person walk the length of the Great Wall of China in a year? If not, how long would it take? (Great Wall of China is 5,500 miles long)

$\frac{5,500}{365}$ = about 15 miles/day. yes could but it would be hard to do it 365 days a row Maybe 2-3 yrs instead.

2- The steps you take in an average year.

answers vary: average 10,000 steps/day $(1 \times 10^4)(4 \times 10^2) = 4 \times 10^6$ days about 4 millio

For Problems 3 – 5 Use the table 3.1

TABLE 3.1 Selected Energy Comparisons

Item	Energy (joules)
Energy released by metabolism of 1 average candy bar	1×10^6
Energy needed for 1 hour of running (adult)	4×10^6
Energy released by burning 1 liter of oil	1.2×10^7
Electrical energy used in an average home daily	5×10^7
Energy released by burning 1 kilogram of coal	1.6×10^9
Energy released by fission of 1 kilogram of uranium-235	5.6×10^{13}
Energy released by fusion of hydrogen in 1 liter of water	6.9×10^{13}
U.S. annual energy consumption	1.0×10^{20}
World annual energy consumption	5.3×10^{20}
Annual energy generation of Sun	1×10^{34}

$\frac{1 \times 10^{34}}{3.2 \times 10^7} = 0.31 \times 10^{27}$ 3.1×10^{26} energy/sec

3- Suppose that we could somehow capture all the energy released by the sun for just 1 second. Would this energy be enough to supply U.S. energy needs for a year? (yes)

seconds in a year $365 \times 24 \times 60 \times 60 = 31,536,000 = 3.2 \times 10^7$

4- How many liters of oil are required to supply the electrical energy needs of an average home for a month? $(5 \times 10^7)(3 \times 10^1) = 15 \times 10^8$ $\frac{1.5 \times 10^9}{1.2 \times 10^7} = 1.25 \times 10^2$ 125 Liters

$1.2 \times 10^7 = 1$ Liter oil / month

5- If you could generate energy by fusing the hydrogen in water, how much water would you need to supply all the energy currently consumed worldwide in one year?

6.9×10^{13} 5.3×10^{20} $\frac{5.3 \times 10^{20}}{6.9 \times 10^{13}} = .77 \times 10^7 = 7.7 \times 10^6$ about 7.7 million

6- The fastest spaceships launched to date are traveling away from Earth at speeds of about 50,000 km per hour. How long would such a spaceship take to reach Alpha Centauri?

$(1\ light - year = 9.5 \times 10^{12}\ km.$

3 stars of the Alpha Centauri system is about 4.3 light years away.)

50,000 km/hr 5×10^4 km/hr ; $4.3(9.5 \times 10^{12})$ 4.1×10^{13}

$\frac{4.1 \times 10^{13}}{5 \times 10^4} = 8.2 \times 10^8$ hours / 8760 hours/year $= 9.3 \times 10^4$ about 93,000 years

7- In 2011, Walmart had profits of $16.4 billion. Express this in terms of dollars per minute.

$\frac{16.4 \times 10^9}{5.3 \times 10^5} = 3.1 \times 10^4$ minutes in a year $365 \times 24 \times 60$ 5.3×10^5

(31,000 per minute)

Section 3E How numbers can Deceive

8-

Jeter and Justice. The following table shows the number of hits (H), number of at-bats (AB), and batting average (AVG = H/AB) for major leaguers Derek Jeter and David Justice in 1995 and 1996.

	1995	1996
Jeter	12 H, 48 AB, AVG = 250	183 H, 582 AB, AVG = 314
Justice	104 H, 411 AB, AVG = 253	45 H, 140 AB, AVG = 321

Source: Ken Ross, *A Mathematician at the Ballpark: Odds and Probabilities for Baseball Fans*, Pi Press, 2004.

a. Which player had the higher batting average in both 1995 and 1996?

b. Compute the batting average for each player for the two years combined.

c. Which player had the higher combined batting average for 1995 and 1996?

d. Explain the apparent inconsistency in these results.

a) Justice

b) Justice $\frac{104+45}{411+140} = \frac{149}{551} = 27\%$

Jeter $\frac{12+183}{48+582} = \frac{195}{630} = 31\%$

c) Justice d) more at bats for Jeter

9-

Airline Arrivals. The following table shows real arrival data for two airlines in five cities (airline names have been changed).

	Excelsior Airlines		Paradise Airlines	
Destination	% On Time	Number of Arrivals	% On Time	Number of Arrivals
Los Angeles	88.9	559	85.6	811
Phoenix	94.8	233	92.1	5255
San Diego	91.4	232	85.5	448
San Francisco	83.1	605	71.3	449
Seattle	85.8	2146	76.7	262
Total		3775		7225

Source: *Technical Review* (1994), p. 3845.

a. Which airline has the higher percentage of on-time flights to the five cities?

b. Compute the percentage of on-time flights for the two airlines over all five cities.

c. Explain the apparent inconsistency in these results.

a) Excelsior airlines had higher on-time percentage for each of the 5 cities

b) Averaged overall cities the on-time percentage was 86.7% Excelsior 89.1% Paradise

c) Excelsior higher in each city Paradise higher overall.

10-

Better Drug. Two drugs, A and B, were tested on a total of 2000 patients, half of whom were women and half of whom were men. Drug A was given to 900 patients, and Drug B to 1100 patients. The results appear in the table below.

	Women	Men
Drug A	5 of 100 cured	400 of 800 cured
Drug B	101 of 900 cured	196 of 200 cured

a. Give numerical evidence to support the claim that Drug B is more effective than Drug A.

b. Give numerical evidence to support the claim that Drug A is more effective than Drug B.

c. Which claim do you think makes more sense? Why?

a) Drug B had a higher cure rate for men (98% vs. 50%) and for women (11.2% vs. 5%)

b) Drug A had better cure rate overall (45% vs. 27%)

c) Since the drugs had such different effects on men and women the individual cure rates should be considered

Section 4A Taking Control of Your Finances

Compute the total cost per year of the following pairs of expenses. Then complete the sentence: On an annual basis, the first set of expenses _____ % of the second set of expenses.

11- Ted goes to a club or concert every two weeks and spends an average of $60 each time: He spends $500 a year on car insurance.

yearly he spend $\frac{52}{2}(60) = \$1560$

$\frac{1560}{500} = 3.12$ 312%

Find the monthly interest payments in the following situation. Assume that monthly interest rates are 1/12 of annual interest rates.

12- Brooke's credit card has an annual interest rate of 21% on her unpaid balance, which averages $900.

$\frac{.21}{12}(900) = \$15.75$

Consider the following situations, which each involve two options. Determine which option is less expensive. Are there unstated factors that might affect your decision?

13- You currently drive 300 miles per week in a car that gets 15 miles per gallon of gas. You are considering buying a new fuel-efficient car for $12,000 (after trade-in on your current car) that gets 50 miles per gallon. Insurance premiums for the new and old car are $800 and $600 per year, respectively. You anticipate spending $1200 per year on repairs for the old car and having no repairs on the new car. Assume gas costs $3.50 per gallon. Over a five-year period, is it less expensive to keep your old car or buy the new car?

New	Old	300 mi/wk
12,000	$1200 repairs	$3.50 gallon
50 mpg	$600	
$800		

Insurance: $4000 | $3,000

$12,000 | $6,000

$(1092)(5) = 5460$ gas | $3640 \times 5 = \$18,200$ gas

New Car $21,460 | Old 27,200

14- If you stay in your home town, you can go to Concord College at a reduced tuition of $3000 per year and pay $800 per month in rent. Or you can leave home, go to Versalia College on a $10,000 scholarship (per year), pay $16,000 per year in tuition, and pay $350 per month to live in a dormitory. You will pay $2000 per year to travel back and forth from Versalia College. Assuming all other factors are equal, which is the less expensive choice on an annual (12-month) basis?

Concord	Versalia
3000	−10,000
$800(12) = 9600$	16,000
	$350(12) = 4200$
	2000

Concord: $12,600 | Versalia: $12,200

15- **Solar Payback Period.** Julie is considering installing photovoltaic panels on the roof of her house. Her monthly electricity bills currently average $85. The cost of installing a photovoltaic system is $18,400; however, she expects to see a 40% reduction in this cost due to tax credits and local rebates. Assuming all of her electrical needs are met by the new system and neglecting possible revenue when the system puts electricity back into the grid, what is the approximate payback period on the investment?

$18,400(.60) = \$11,040$ cost of system

$\frac{11,040}{85} = 130$ months

or 10.8 years

Section 4B The Power of Compounding

16- What is the difference between simple interest and compound interest? Why do you end up with more money with compound interest? Simple Interest is interest on principal only. Compound interest you get interest on interest.

17- An account with an APR of 4% and quarterly compounding increases in value every three months by

a. 1%. b. 1/4%. c. 4%.

18-

The annual percentage yield (APY) is always

a. less than the APR.

b. at least as great as the APR.

c. the same as the APR.

Compound Interest Formula:

$$A = P \times \left(1 + \frac{\text{APR}}{n}\right)^{(nY)}$$

Compute the balance in the following accounts after the stated period of time.

19- $2,000 is invested for 5 years with an APR of 3% and daily compounding.

$2000\left(1+\frac{.03}{365}\right)^{5(365)} = \$2,323.65$

20- $15,000 is invested for 15 years with an APR of 4.2% and monthly compounding.

$15{,}000\left(1+\frac{.042}{12}\right)^{15(12)} = \$28,133.20$

How much must you deposit today into the following accounts in order to have $25,000 in 8 years for a down payment on a house? Assume no additional deposits are made.

21- An account quarterly compounded and an APR of 4.5%. $25{,}000 = P\left(1+\frac{.045}{4}\right)^{4(8)}$

$17,477.00

22- An account daily compounded and an APR of 4%. $25{,}000 = P\left(1+\frac{.04}{365}\right)^{8(365)}$

$18,154.04

23- How long will it take your money to triple at an APR of 8% compounded annually?

14.3 years

$3P = P\left(1+\frac{.08}{1}\right)^{y}$

$3 = 1.08^{y} \quad y = \log_{1.08} 3$

24-

Retirement Fund. A retired couple plans to supplement their Social Security with interest earned by a $120,000 retirement fund.

a. If the fund compounds interest monthly at an annual rate of 6%, which the couple takes out and spends each month, how much interest is generated each month?

a) $120{,}000\left(1+\frac{.06}{12}\right)^{12(1)} = 120{,}600$

$600

b. Suppose the annual interest rate suddenly drops to 3%. What is the resulting interest payment each month?

b) $300

c. Estimate the annual interest rate needed to generate $900 each month in interest.

c) $120{,}900 = 120{,}000\left(1+\frac{APR}{12}\right)^{12(1)}$

9%

Section 4C Savings Plans and Investments

$$A = \text{PMT} \times \frac{\left[\left(1 + \frac{\text{APR}}{n}\right)^{(nY)} - 1\right]}{\left(\frac{\text{APR}}{n}\right)}$$

$$\text{total return} = \frac{(A - P)}{P} \times 100\%$$

$$\text{annual return} = \left(\frac{A}{P}\right)^{1/Y} - 1$$

25- Find the savings plan balance after 5 years with an APR of 2.5% and monthly payments of $100.

$$100\left[\frac{\left(1 + \frac{.025}{12}\right)^{12(5)} - 1}{\frac{.025}{12}}\right] = \$6384.05$$

26- A friend has an IRA with an APR of 6.25%. She started the IRA at age 25 and deposits $50 a month. How much will her IRA contain when she retires at age 65? Compare that amount to the total deposits made over the time period.

$$50\left[\frac{\left(1 + \frac{.0625}{12}\right)^{12(40)} - 1}{\left(\frac{.0625}{12}\right)}\right] = \$106,595.63$$

$\$50 \cdot 40 \text{ yrs} \cdot 12 \text{ mnths} = \$24,000$

27- At age 35, you start saving for retirement. If your investment plan pays an APR of 6% and you want to have $2 million when you retire in 30 years. How much should you deposit monthly? (your employer will match what you deposit)

$$2,000,000 = \text{PMT} \cdot \left[\frac{\left(1 + \frac{.06}{12}\right)^{12(30)} - 1}{\left(\frac{.06}{12}\right)}\right]$$

$\text{PMT} = \$1,991$

Compute the total and annual returns on the following investments

28- You pay $8,000 for a municipal bond. When it matures after 20 years, you receive $12,500.

$$\frac{12,500 - 8,000}{8,000} = 56.3\%$$

$$\left(\frac{12,500}{8,000}\right)^{1/20} - 1 = 2.3\%$$

29- Five years after paying $5000 for shares in a new company, you sell the shares for $3000 (at a loss).

$$\frac{3,000 - 5,000}{5,000} = -40\% \qquad \left(\frac{3,000}{5,000}\right)^{\frac{1}{5}} - 1 = -9.7\%$$

Section 4D Loan Payments, Credit Cards, and Mortgages

$$\text{PMT} = \frac{P \times \frac{\text{APR}}{n}}{\left[1 - \left(1 + \frac{\text{APR}}{n}\right)^{(-nY)}\right]}$$

30-

In the early years of a 30-year mortgage loan,

a. most of the payment goes to the principal.

b. most of the payment goes to interest.

c. equal amounts go to principal and interest.

31-

A $120,000 loan with $500 in closing costs plus 1 point requires an advance payment of

a. $1500.

b. $1700.

c. $500.

$500 + 1200

32 – 33

a. Calculate the monthly payment.

b. Determine the total amount paid over the term of the loan.

c. Of the total amount paid, what percentage is paid toward the principal and what percentage is paid for interest?

32- A student loan of $12,000 at a fixed APR of 7% for 10 years.

$$\frac{12{,}000\left(\frac{.07}{12}\right)}{1-\left(1+\frac{.07}{12}\right)^{-10(12)}} = 70$$

a) $139.33

b) $16719.60

c) $\frac{12{,}000}{16719.60} = 71.8\%$; 28.2%

33- A home mortgage of $300,000 at a fixed APR of 4% for 30 years.

$$\frac{300{,}000\left(\frac{.04}{12}\right)}{1-\left(1+\frac{.04}{12}\right)^{-30(12)}} = \frac{1{,}000}{0.698} = \$1432.25$$

a)

b) $515,610.00

c) 58%, 42%

$\frac{300{,}000}{515{,}610}$; $\frac{215{,}610}{515{,}610}$

Principal and Interest. For the following loans, make a table (as in Example 2) showing the amount of each monthly payment that goes toward principal and interest for the first three months of the loan.

34 - A student loan of $24,000 at a fixed APR of 8% for 15 years.

PMT = 229.36

Interest = $\frac{.08}{12}(24{,}000) = \$160.00$

month	Interest	Payment Principal	New Principal
1	$160.00	$69.36	$23,930.64
2	$159.54	69.82	$23,860.82
3	$159.07	70.29	$23,790.53

Credit Card Debt. Suppose that on January 1 you have a balance of $5000 on the following credit cards, which you want to pay off in the given amount of time. Assume that you make no additional charges to the card after January 1.

a. Calculate your monthly payments.

b. When the card is paid off, how much will you have paid since January 1?

c. What percentage of your total payment (part (b)) is interest?

35- The credit card APR is 20%, and you want to pay off the balance in 2 years.

a) $254.48

$$\frac{5{,}000\left(\frac{.20}{12}\right)}{1-\left(1+\frac{.20}{12}\right)^{-2(12)}} = 254.48$$

b) $6107.52

c) 18.1% $\Rightarrow \frac{1107.52}{6107.52}$

Comparing Loan Options. Compare the monthly payments and total loan costs for the following pairs of loan options. Assume that both loans are fixed rate and have the same closing costs. Discuss the pros and cons of each loan.

36- You need a $180,000 loan.

Option 1: a 30 – year loan at an APR of 7.25%

Option 2: a 15 – year loan at 6.8%

Option 1: $1227.92 month

$442,051.20 Total

$$\frac{180{,}000\left(\frac{.0725}{12}\right)}{1-\left(1+\frac{.0725}{12}\right)^{-30(12)}}$$

Option 2

$1597.83 month

$287,609.40 Total

Closing Costs. Consider the following pairs of loan options for a $120,000 mortgage. Calculate the monthly payment and total closing costs for each option. Explain which option you would choose and why.

37- Choice 1: 30 – year fixed rate at 3.5% with ~~no~~ closing costs of $1,000 and no points

Choice 2: 30 – year fixed rate at 3% with no closing costs of $1,500 and 4 points

Choice 1: $538.55 month $1,000 closing

Choice 2: 505.92 month $6300 closing

Save $32.63 month $5300 difference Closing

$\frac{5300}{32.63} = 162$ months

stay in house for over 13.5 years take lower interest

38-

Accelerated Loan Payment. Suppose you have a student loan of $60,000 with an APR of 8% for 25 years.

a. What are your required monthly payments?

b. Suppose you would like to pay the loan off in 15 years instead of 25. What monthly payments will you need to make?

c. Compare the total amounts you'll pay over the loan term if you pay the loan off in 25 years versus 15 years.

a) $463.09

b) $573.39

c) $138,927.00 25 yrs.

$103,210.20 15 yrs.