Pre-Calculus
Chapter 6 Review

Name: Key
Period: _____

Solve the following system by substitution:

1- $\begin{cases} x - y = 0 \\ 5x - 3y = 10 \end{cases}$

$x = y$
$5(y) - 3y = 10$
$2y = 10$
$y = 5$
$(5, 5)$

2- $\begin{cases} \frac{1}{5}x + \frac{1}{2}y = 7 \\ x + y = 20 \end{cases}$

$x = 20 - y$
$\frac{1}{5}(20 - y) + \frac{1}{2}y = 7$
$4 - \frac{1}{5}y + \frac{1}{2}y = 7$
$4 + \frac{3}{10}y = 7$
$\frac{3}{10}y = 3$
$y = 10$
$(10, 10)$

3- $\begin{cases} y = 4 - x^2 \\ y = x^2 - 4 \end{cases}$

$4 - x^2 = x^2 - 4$
$4 = 2x^2 - 4$
$8 = 2x^2$
$4 = x^2$
$x = \pm 2$
$(2, 0), (-2, 0)$

Solve the following system by elimination:

4- $\begin{cases} 3x - 2y = 6 \\ -6x + 4y = -12 \end{cases}$ $\times 2$

$6x - 4y = 12$
$0 = 0$ infinite

$3x - 2y = 6$
$3x = 2y + 6$
$x = \frac{2}{3}y + 2$
$(\frac{2}{3}y + 2, y)$

5- $\begin{cases} 5x + 3y = 18 \\ 2x - 7y = -1 \end{cases}$ $\times 2$, $\times -5$

$10x + 6y = 36$
$-10x + 35y = 5$
$41y = 41$
$y = 1$
$5x + 3(1) = 18$
$5x = 15$ $x = 3$
$(3, 1)$

6- $\begin{cases} x^2 + 2y^2 = 2 \\ 2x^2 - 3y = 15 \end{cases}$ $\times -2$

$-2x^2 - 4y^2 = -4$
$-4y^2 - 3y = 11$
$4y^2 + 3y + 11 = 0$
$y = \frac{-3 \pm \sqrt{9 - 4(4)(11)}}{2(8)}$
$= \frac{-3 \pm \sqrt{-167}}{16}$
no sol.

Use the method of back substitution to solve the system:

7- $\begin{cases} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{cases}$

$x - 2(-1) + 3(2) = 9$ $x = 1$
$y + 3(2) = 5$ $y = -1$
$(1, -1, 2)$

Solve the system of linear equations:

8- $\begin{cases} x - 3y + z = 1 \\ 2x - y - 2z = 0 \\ x + 2y - 3z = -1 \end{cases}$ $\times -2$

$(\frac{7}{5}z - \frac{1}{5}, \frac{4}{5}z - \frac{2}{5}, z)$

$-2x + 6y - 2z = -2$
$2x - y - 2z = 0$
$5y - 4z = -2$

$2x - y - 2z = 0$
$-2x - 4y + 6z = 2$
$-5y + 4z = 2$

$5y - 4z = -2$
$-5y + 4z = 2$
$0 = 0$
infinite solutions

$-5y = -4z + 2$
$y = \frac{4}{5}z - \frac{2}{5}$
$x + 2(\frac{4}{5}z - \frac{2}{5}) - 3z = -1$
$x + \frac{8}{5}z - \frac{4}{5} - 3z = -1$
$x - \frac{7}{5}z = -\frac{1}{5}$
$x = \frac{7}{5}z - \frac{1}{5}$

9- $\begin{cases} x - 2y - 3z = 5 \\ 2x + y - z = 5 \\ 4x - 3y - 2z = 5 \end{cases}$ $\times -2$

$-4x - 2y + 2z = -10$
$4x - 3y - 2z = 5$
$-5y = -5$
$y = 1$

$-2x + 4y + 6z = -10$
$2x + y - z = 5$
$5y + 5z = -5$

$5(1) + 5z = -5$
$5z = -10$
$z = -2$

$x - 2(1) - 3(-2) = 5$
$x + 4 = 5$
$x = 1$
$(1, 1, -2)$

10- $\begin{cases} 2x+y-z=-8 \\ -x+y+z=3 \\ -2x+4z=18 \end{cases}$ -1

$2x+y-z=-8$

$x-y-z=-3$

$3x-2z=-11$ ·2

$-2x+4z=18$

$6x-4z=-22$

$4x=-4$

$x=-1$

$-2(-1)+4z=18$

$4z=16$

$z=4$

$1+y+4=3$

$y=-2$

$(-1,-2,4)$

11- $\begin{cases} 2x+3y-12z=1 \\ x-2y+z=4 \\ 4x+y-14z=7 \end{cases}$ -4 -2

$(3z+2, 2z-1, z)$

$-4x-6y+24z=-2$

$4x+y-14z=7$

$-5y+10z=5$

$-y+2z=1$

$-y+2z=1$

$y-2z=-1$

$0=0$

infinite solutions

$-4x+8y-4z=-16$

$4x+y-14z=7$

$9y-18z=-9$

$y-2z=-1$

$y=2z-1$

$x-2(2z-1)+z=4$

$x-4z+2+z=4$

$x-3z+2=4$

$x=3z+2$

Find the partial fractions for each of the following:

12. $\frac{(3x+1)}{(x^2-2x-15)}$

$\frac{3x+1}{(x-5)(x+3)}=\frac{A}{(x-5)}+\frac{B}{(x+3)}=\frac{2}{x-5}+\frac{1}{x+3}$

$3x+1=A(x+3)+B(x-5)$

5 $(A+B=3)$ $5A+5B=15$

$3A-5B=1$ $3A-5B=1$

$8A=16$

$A=2$

$B=1$

13. $\frac{(x+4)}{(x^3-x^2+4x-4)}=\frac{x+4}{x^2(x-1)+4(x-1)}=\frac{x+4}{(x^2+4)(x-1)}$

$\frac{x+4}{(x^2+4)(x-1)}=\frac{Ax+B}{x^2+4}+\frac{C}{x-1}=\frac{-x}{x^2+4}+\frac{1}{x-1}$

$x+4=(Ax+B)(x-1)+C(x^2+4)$

$x+4=Ax^2-Ax+Bx-B+Cx^2+4C$

$0=A+C$; $1=-A+B$; $4=-B+4C$

$0=A+C$

$1=-A+B$ $\rightarrow B+C=1$ $\rightarrow 1=B+C$

$C=5C$

$C=1$

$B=0$

$A=-1$

14. $\frac{3x}{(x-2)^2}$

$\frac{3x}{(x-2)^2}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}=\frac{3}{x-2}+\frac{6}{(x-2)^2}$

$3x=A(x-2)+B$

$A=3$

$0=-2A+B$

$0=-2(3)+B$

$B=6$

15. $\frac{(-x+10)}{(x^2+x-12)}$

$=\frac{-x+10}{(x+4)(x-3)}=\frac{A}{x+4}+\frac{B}{x-3}$

$-x+10=A(x-3)+B(x+4)$

3 $(-1=A+B)$

$10=-3A+4B$

$-3=+3A+3B$

$7=7B$

$B=1$; $A=-2$

$=\frac{-2}{x+4}+\frac{1}{x-3}$

16. $\frac{(2x-4)}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$

$2x-4=A(x-1)^2+B(x)(x-1)+Cx$

$2x-4=A(x^2-2x+1)+B(x^2-x)+Cx$

$0=A+B$ $\Rightarrow$ $0=-4+B$

$B=4$

$2=-2A-B+C$

$-4=A$

$2=-2(-4)-4+C$

$-2=C$

$\frac{-4}{x}+\frac{4}{(x-1)}+\frac{-2}{(x-1)^2}$

17. $\frac{(2x-1)}{(x^3+x)}=\frac{2x-1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$2x-1=A(x^2+1)+x(Bx+C)$

Bx^2+Cx

$0=A+B$

$2=C$

$-1=A$

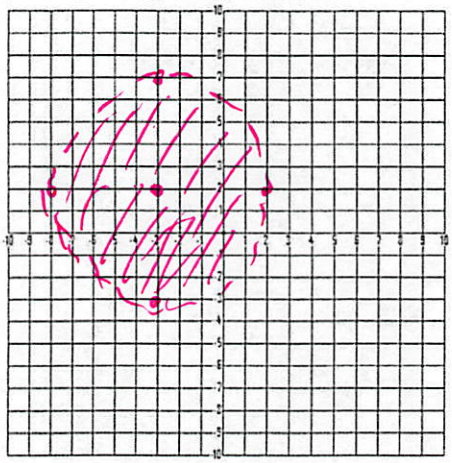
$1=B$

$-\frac{1}{x}+\frac{x+2}{x^2+1}$

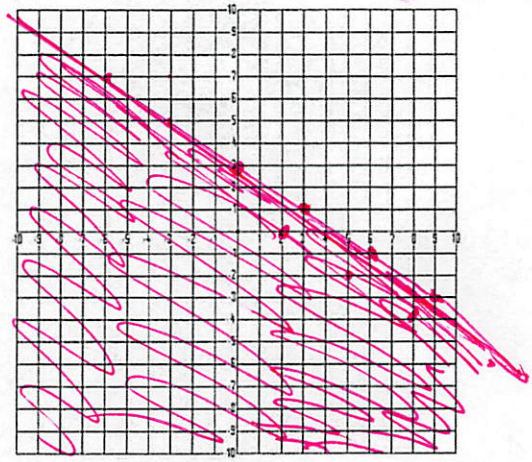
Graph the following inequalities. If the region is bounded find the coordinates of the vertices.

18. $(x+3)^2 + (y-2)^2 < 25$

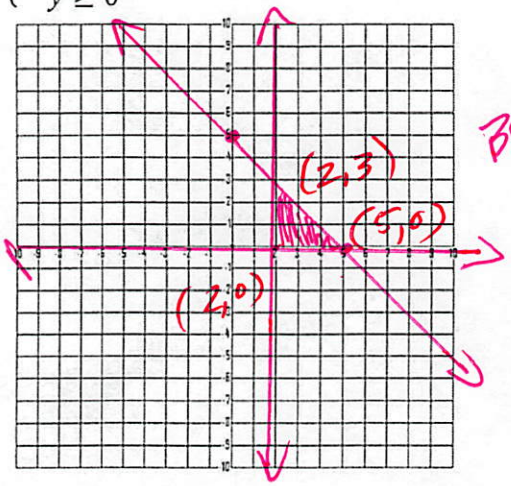
C(-3,2)
r = 5



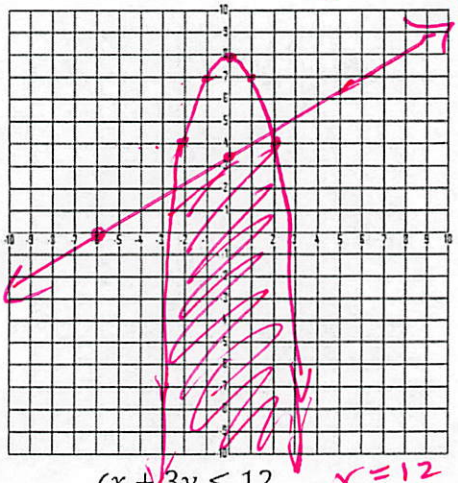
19. $2x + 3y \leq 6$ x = 3 y = 2



20. $\begin{cases} x + y \leq 5 \\ x \geq 2 \\ y \geq 0 \end{cases}$

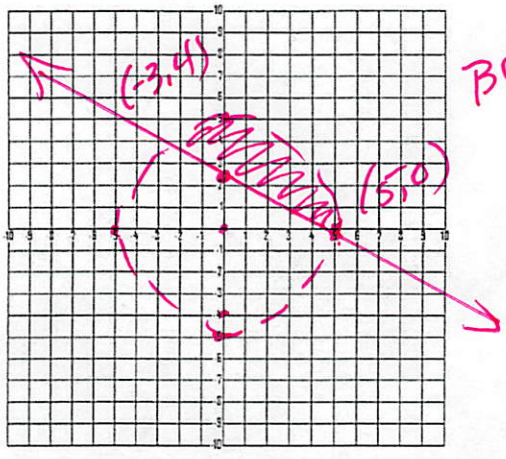


21. $\begin{cases} x^2 + y \leq 8 \\ x - 2y \geq -6 \end{cases}$ $y \leq -x^2 + 8$ x = -6 y = 3

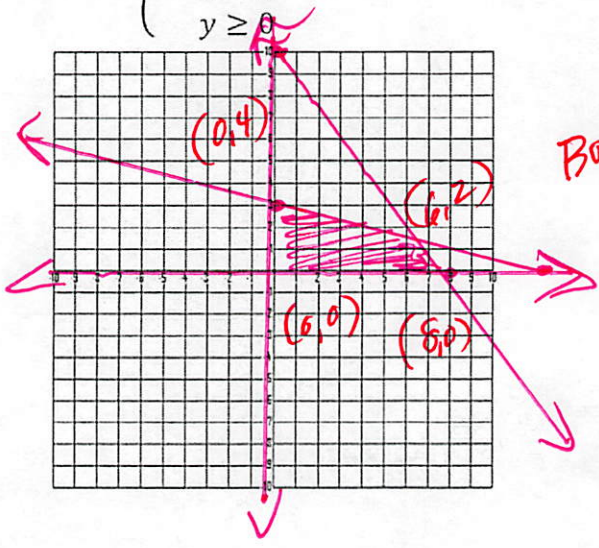


22. $\begin{cases} x^2 + y^2 < 25 \\ x + 2y \geq 5 \end{cases}$ x = 5 y = 5/2

C(0,0)
r = 5



23. $\begin{cases} x + 3y \leq 12 \\ x + y \leq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$ x = 12 y = 4



24. The admission fee into a movie theatre is $4.00 for children and $8.50 for adults. On a certain day, 1800 people went to the theatre, and admissions fees collected totaled $13,050. How many children and how many adults were admitted?

x = # children
y = # adults

$4x + 8.5y = 13{,}050$
$-4(x + y = 1800) \Rightarrow$

$4x + 8.5y = 13{,}050$
$-4x - 4y = -7200$

$4.5y = 5850$
$y = 1300$

500 children
1300 adults

25. A man invests his savings in two accounts, one paying 6% interest per year and the other paying 7%. He has twice as much invested in the 7% account as in the 6% account, and his annual interest income is $600. How much is invested in each account?

x: 6% account
y: 7% account

$y = 2x$
$.06x + .07y = 600$
$.06x + .07(2x) = 600$
$.06x + .14x = 600$
$.2x = 600$
$x = 3{,}000$

3,000; 6%
6,000; 7%

26. A piggy bank contains 50 coins, all of them nickels, dimes or quarters. The total value of the coins is $5.60, and the value of the dimes is five times the value of the nickels. How many coins of each type are there?

x: nickels
y: dimes
z: quarters

$x + y + z = 50$
$20(.05x + .1y + .25z = 5.60) \Rightarrow x + 2y + 5z = 112$

$(x + y + z = 50)(-5)$
$-5x - 5y - 5z = -250$

$-4x - 3y = -138$

$y = 2.5x$

$-4x - 3(2.5x) = -138$
$-4x - 7.5x = -138$
$-11.5x = -138$; $x = 12$

$y = 2.5(12)$
$= 30$

12 nickels
30 dimes
8 quarters

27. Anne, Barry, and Cathy enter a bakery. Anne orders one juice, two muffins and two donuts, and pays $6.25. Barry orders one muffin and three donuts, and pays $3.75. Cathy orders one juice, three muffins, and four donuts, and pays $9.25. Find the price of juice, muffins, and donuts at this bakery.

x: juice
y: muffin
z: donuts

$x + 2y + 2z = 6.25 \Rightarrow x + 2y + 2z = 6.25$
$y + 3z = 3.75$
$-1(x + 3y + 4z = 9.25) \Rightarrow -x - 3y - 4z = -9.25$

$-y - 2z = -3$
$y + 3z = 3.75$

$z = .75$

$y + 3(.75) = 3.75$
$y + 2.25 = 3.75$
$y = 1.50$

$x + 2(1.5) + 2(.75) = 6.25$
$x + 3 + 1.5 = 6.25$
$x = 1.75$

Juice $1.75
muffin $1.50
Donut $.75