

SM3

Factor the common factor out of each expression.

1) $35x^2y + 7xy$

2) $6y^2 + 9yx$

Factor each completely.

3) $x^2 + 9x + 20$

4) $6x^2 + 36x - 42$

5) $5x^2 + 44x + 63$

6) $2n^2 + 3n - 5$

7) $25x^2 - 9$

8) $64x^2 - 32x + 4$

9) $4b^2 + 4b + 1$

10) $18n^2 - 2$

11) $125x^3 - 216$

12) $16x^3 + 2$

13) $16r^3 - 56r^2 - 14r + 49$

14) $15x^3 - 9x^2 - 5x + 3$

Evaluate each function using synthetic division and the remainder theorem.

15) $f(x) = x^3 - 11x^2 + 35x - 30$ at $f(5)$

16) $f(x) = 6x^5 + 35x^4 + 24x^3 - 3x^2 + 14x + 20$ at $f(-5)$

17) $f(x) = x^5 - 4x^4 - 18x^3 + 39x^2 - 23x + 29$ at $f(6)$

18) $f(x) = x^4 + 7x^3 + 17x^2 + 10x - 24$ at $f(-3)$

Given a polynomial and one of it's factors, find the remaining factors.

19) $f(x) = x^3 - 10x^2 + 29x - 20$; $x - 5$

20) $f(x) = x^3 + 3x^2 - 10x - 24$; $x + 2$

21) $f(x) = x^3 - 3x^2 - 4x + 12$; $x - 3$

22) $f(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$; $x + 2$

Given a polynomial and one of it's zeros.

Find the remaining zeros, including irrational and imaginary.

23) $f(x) = x^4 + 3x^3 - 9x^2 + 3x - 10$; 2

24) $f(x) = x^4 + 6x^3 + 2x^2 - 18x - 15$; -5

25) $f(x) = x^4 - 4x^3 - 3x^2 + 16x - 4$; -2

26) $f(x) = x^4 - 4x^2 + 12x - 9$; -3

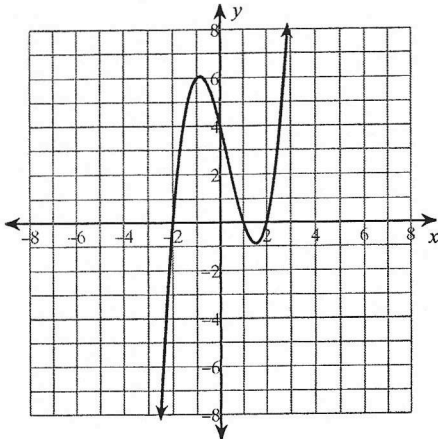
In each of the following, use the graph to identify the x-intercepts, zeros, factors, and the equation of the function.

27) x-intercepts:

zeros:

factors:

equation:

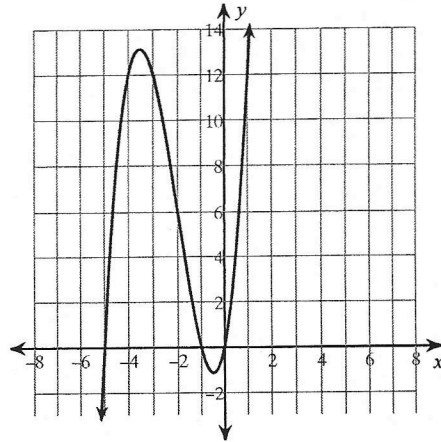


28) x-intercepts:

zeros:

factors:

equation:

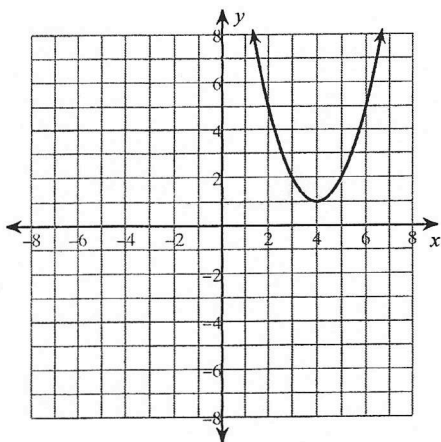


State the number of real zeros and the number of imaginary zeros using the function and the graph.

29) $f(x) = x^2 - 8x + 17$

real zeros:

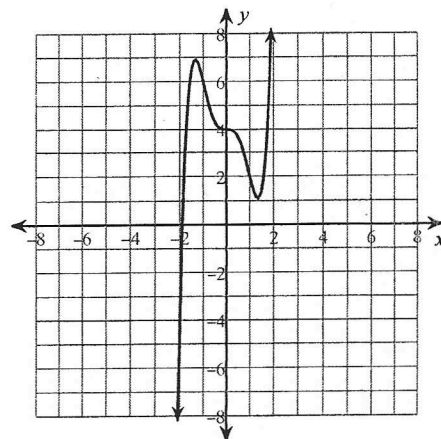
imaginary zeros:



30) $f(x) = x^5 - 3x^3 + 4$

real zeros:

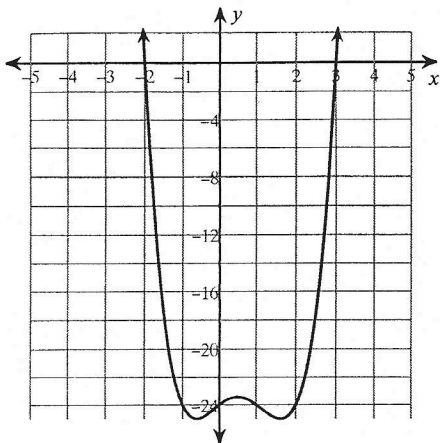
imaginary zeros:



Given a polynomial and its graph. Find the all the zeros, rational, irrational and imaginary.

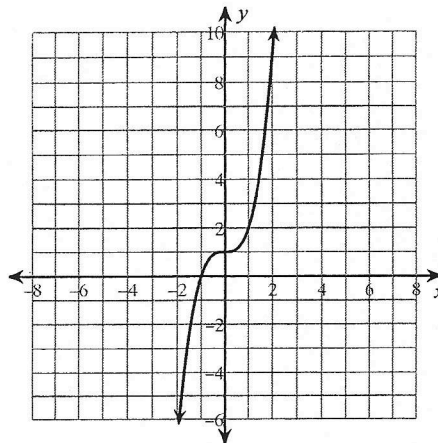
31) $f(x) = x^4 - 2x^3 - x^2 + 2x - 24$

zeros:



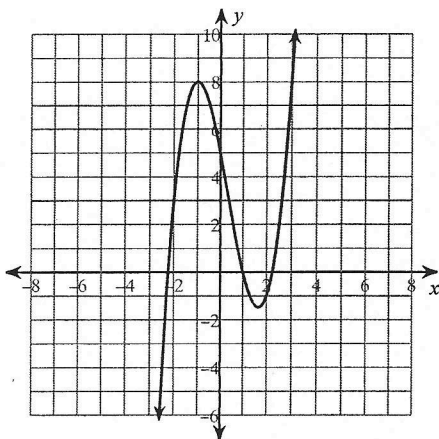
32) $f(x) = x^3 + 1$

zeros:



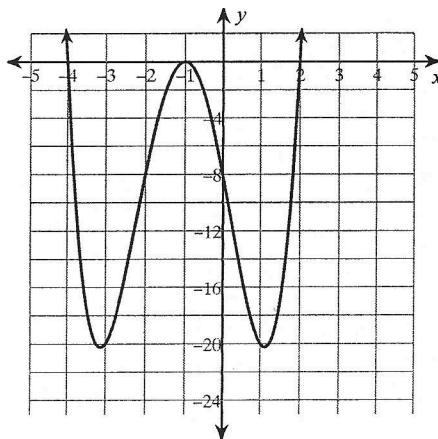
33) $f(x) = x^3 - x^2 - 5x + 5$

zeros:



34) $f(x) = x^4 + 4x^3 - 3x^2 - 14x - 8$

zeros:



Write a polynomial function of least degree with integral coefficients that has the given zeros.

35) 1, -4, -2, 3

36) -5, $\sqrt{3}$

37) 0, -1, i

38) 4, $-2i$

SM3

Factor the common factor out of each expression.

1) $35x^2y + 7xy$

$$7xy(5x+1)$$

2) $6y^2 + 9yx$

$$3y(2y+3x)$$

Factor each completely.

3) $x^2 + 9x + 20$

$$(x+5)(x+4)$$

5) $5x^2 + 44x + 63$

$$(5x+9)(x+7)$$

| | | |
|----|-----------------|-----|
| | x | 7 |
| 5x | 5x ² | 35x |
| 9 | 9x | 63 |

$$\frac{35x^2}{9x} \quad 35x$$

7) $25x^2 - 9$

$$(5x+3)(5x-3)$$

9) $4b^2 + 4b + 1$

$$(2b+1)^2$$

11) $125x^3 - 216$

$a = 5x; b = 6$

$$(5x-6)(25x^2+30x+36)$$

4) $6x^2 + 36x - 42$

$$6(x^2+6x-7)$$

$$6(x+7)(x-1)$$

6) $2n^2 + 3n - 5$

$$(2n+5)(n-1)$$

| | | |
|----|-----------------|-----|
| | n | -1 |
| 2n | 2n ² | -2n |
| 5 | 5n | -5 |

$$\frac{-10n^2}{5n} \quad -2n$$

8) $64x^2 - 32x + 4$

$$4(16x^2 - 8x + 1)$$

$$4(4x-1)^2$$

10) $18n^2 - 2$

$$2(9n^2 - 1)$$

$$2(3n+1)(3n-1)$$

12) $16x^3 + 2$

$a = 2x, b = 1$

$$2(8x^3+1)$$

$$2(2x+1)(4x^2-2x+1)$$

13) $16r^3 - 56r^2 - 14r + 49$

$$8r^2(2r-7) - 7(2r-7)$$

$$(8r-7)(8r^2-7)$$

14) $15x^3 - 9x^2 - 5x + 3$

$$3x^2(5x-3) - 1(5x-3)$$

$$(5x-3)(3x^2-1)$$

Evaluate each function using synthetic division and the remainder theorem.

15) $f(x) = x^3 - 11x^2 + 35x - 30$ at $f(5)$

$$\begin{array}{r|rrrr} 5 & 1 & -11 & 35 & -30 \\ & & 5 & -30 & 25 \\ \hline & 1 & -6 & 5 & -5 \end{array}$$

$f(5) = -5$

16) $f(x) = 6x^5 + 35x^4 + 24x^3 - 3x^2 + 14x + 20$ at $f(-5)$

$$\begin{array}{r|rrrrrr} -5 & 6 & 35 & 24 & -3 & 14 & 20 \\ & & -30 & -25 & 5 & -10 & -20 \\ \hline & 6 & 5 & -1 & 2 & 4 & 0 \end{array}$$

$f(-5) = 0$

17) $f(x) = x^5 - 4x^4 - 18x^3 + 39x^2 - 23x + 29$ at $f(6)$

$$\begin{array}{r|rrrrrr} 6 & 1 & -4 & -18 & 39 & -23 & 29 \\ & & 6 & 12 & -36 & 18 & -30 \\ \hline & 1 & 2 & -6 & 3 & -5 & -1 \end{array}$$

$f(6) = -1$

18) $f(x) = x^4 + 7x^3 + 17x^2 + 10x - 24$ at $f(-3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 7 & 17 & 10 & -24 \\ & & -3 & -12 & -15 & +15 \\ \hline & 1 & 4 & 5 & -5 & -9 \end{array}$$

$f(-3) = -9$

Given a polynomial and one of its factors, find the remaining factors.

19) $f(x) = x^3 - 10x^2 + 29x - 20$; $x - 5$

$$\begin{array}{r|rrrr} 5 & 1 & -10 & 29 & -20 \\ & & 5 & -25 & 20 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$x^2 - 5x + 4 = (x-1)(x-4)$

20) $f(x) = x^3 + 3x^2 - 10x - 24$; $x + 2$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -10 & -24 \\ & & -2 & -2 & 24 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$x^2 + x - 12 = (x+4)(x-3)$

21) $f(x) = x^3 - 3x^2 - 4x + 12$; $x - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & -4 & 12 \\ & & 3 & 0 & -12 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$x^2 - 4 = (x+2)(x-2)$

22) $f(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$; $x + 2$

$$\begin{array}{r|rrrrr} -2 & 1 & 6 & 7 & -6 & -8 \\ & & -2 & -8 & 2 & 8 \\ \hline & 1 & 4 & -1 & -4 & 0 \\ & & 1 & 5 & 4 & \\ \hline & 1 & 5 & 4 & 0 & \end{array}$$

$x^2 + 5x + 4 = (x+4)(x+1)(x-1)$

Given a polynomial and one of its zeros.

Find the remaining zeros, including irrational and imaginary.

23) $f(x) = x^4 + 3x^3 - 9x^2 + 3x - 10$; 2

$$\begin{array}{r|rrrrr} 2 & 1 & 3 & -9 & 3 & -10 \\ & & 2 & 10 & 2 & 10 \\ \hline & 1 & 5 & 1 & 5 & 0 \\ & & -5 & 0 & -5 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$x^2 + 1 = (x-i)(x+i)$

24) $f(x) = x^4 + 6x^3 + 2x^2 - 18x - 15$; -5

$$\begin{array}{r|rrrrr} -5 & 1 & 6 & 2 & -18 & -15 \\ & & -5 & -5 & 15 & 15 \\ \hline & 1 & 1 & -3 & -3 & 0 \\ & & 1 & 0 & -3 & \\ \hline & 1 & 0 & -3 & 0 & \end{array}$$

$x^2 - 3 = (x-\sqrt{3})(x+\sqrt{3})$

25) $f(x) = x^4 - 4x^3 - 3x^2 + 16x - 4$; -2

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -3 & 16 & -4 \\ & & -2 & 12 & -18 & 4 \\ \hline & 1 & -6 & 9 & -2 & 0 \\ & & 2 & -8 & 2 & \\ \hline & 1 & -4 & 1 & 0 & \end{array}$$

$x^2 - 4x + 1 = (x-2) \pm \sqrt{3}$

26) $f(x) = x^4 - 4x^2 + 12x - 9$; -3

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -4 & 12 & -9 \\ & & -3 & 9 & -15 & 9 \\ \hline & 1 & -3 & 5 & -3 & 0 \\ & & 1 & -2 & 3 & \\ \hline & 1 & -2 & 3 & 0 & \end{array}$$

$x^2 - 2x + 3 = (x-1) \pm i\sqrt{2}$

$\frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$\frac{2 \pm \sqrt{4 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2i\sqrt{2}}{2}$

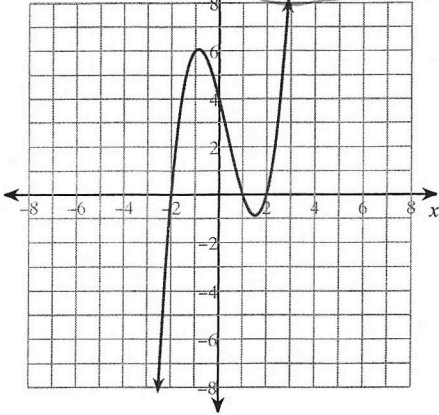
In each of the following, use the graph to identify the x-intercepts, zeros, factors, and the equation of the function.

27) x-intercepts: $(-2, 0), (1, 0), (2, 0)$

zeros: $-2, 1, 2$

factors: $(x+2)(x-1)(x-2)$

equation: $(x^2-4)(x-1)$
 $= x^3 - x^2 - 4x + 4$

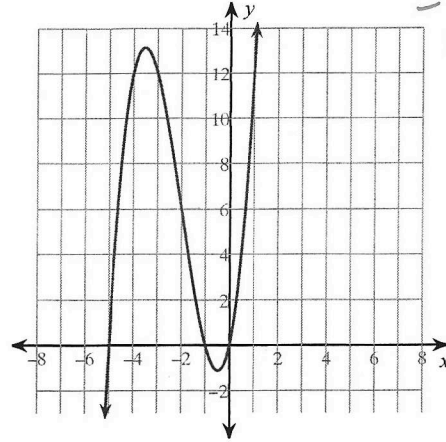


28) x-intercepts: $(-5, 0), (-1, 0), (0, 0)$

zeros: $-5, -1, 0$

factors: $(x+5)(x+1)x$

equation: $(x^2+6x+5)x$
 $= x^3 + 6x^2 + 5x$

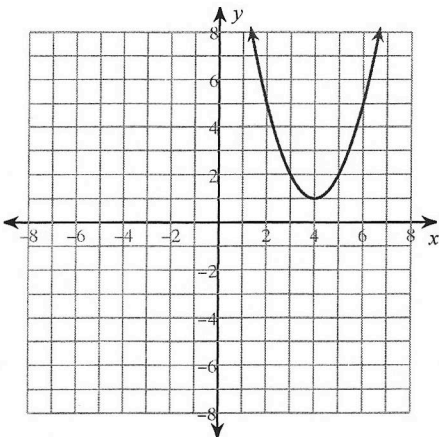


State the number of real zeros and the number of imaginary zeros using the function and the graph.

29) $f(x) = x^2 - 8x + 17$

real zeros: *none*

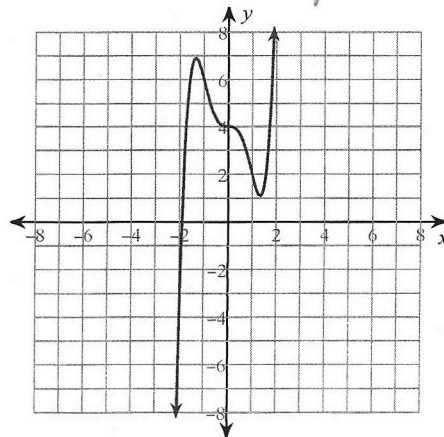
imaginary zeros: *2*



30) $f(x) = x^5 - 3x^3 + 4$

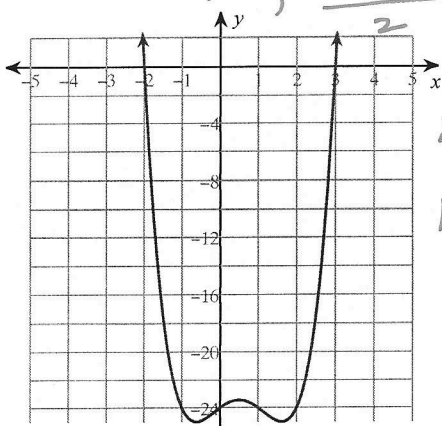
real zeros: *1*

imaginary zeros: *4*



Given a polynomial and its graph. Find the all the zeros, rational, irrational and imaginary.

31) $f(x) = x^4 - 2x^3 - x^2 + 2x - 24$
 zeros: $-2, 3, \frac{1 \pm i\sqrt{15}}{2}$



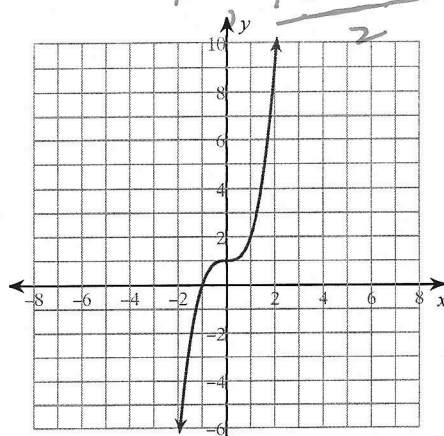
$$x^2 - x + 4$$

$$\frac{1 \pm \sqrt{1 - 4(4)}}{2}$$

$$\frac{1 \pm \sqrt{-15}}{2}$$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -1 & 2 & -24 \\ & & -2 & 8 & -14 & 24 \\ \hline 3 & 1 & -4 & -7 & -12 & 0 \\ & & 3 & -3 & 12 & 0 \\ \hline 1 & 1 & -3 & 4 & 0 & \end{array}$$

32) $f(x) = x^3 + 1$
 zeros: $-1, \frac{1 \pm i\sqrt{3}}{2}$



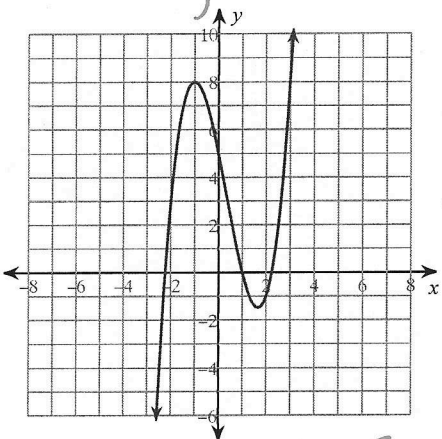
$$x^2 - x + 1$$

$$\frac{1 \pm \sqrt{1 - 4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & & -1 & 1 & -1 \\ \hline 1 & 1 & -1 & 1 & 0 \end{array}$$

33) $f(x) = x^3 - x^2 - 5x + 5$
 zeros: $1, \pm\sqrt{5}$

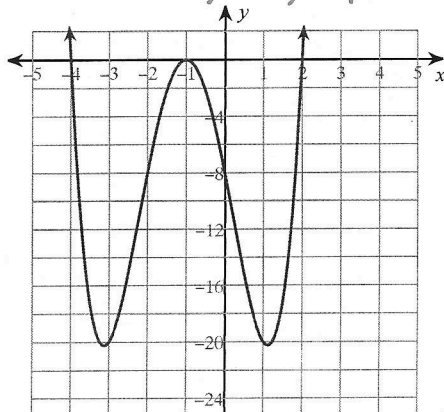


$$x^2 - 5$$

$$x = \pm\sqrt{5}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -5 & 5 \\ & & 1 & 0 & -5 \\ \hline 1 & 1 & 0 & -5 & 0 \end{array}$$

34) $f(x) = x^4 + 4x^3 - 3x^2 - 14x - 8$
 zeros: $-4, 2, -1$ mult 2



$$x^2 + 2x + 1$$

$$(x+1)(x+1)$$

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & -3 & -14 & -8 \\ & & -4 & 0 & 12 & 8 \\ \hline 2 & 1 & 0 & -3 & -2 & 0 \\ & & 2 & 4 & 2 & 0 \\ \hline 1 & 1 & 2 & 1 & 0 & \end{array}$$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

35) 1, -4, -2, 3
 $(x-1)(x+4)(x+2)(x-3)$
 $(x^2+3x-4)(x^2-x-6)$
 $x^4 - x^3 - 6x^2 + 3x^2 - 3x^2 - 18x - 4x^2 + 4x + 24$

$$x^4 + 2x^3 - 13x^2 - 14x + 24$$

36) $-5, \sqrt{3}$
 $(x+5)(x-\sqrt{3})(x+\sqrt{3})$
 $(x+5)(x^2-3)$

$$x^3 + 5x^2 - 3x - 15$$

37) 0, -1, i
 $x(x+1)(x+i)(x-i)$
 $(x^2+x)(x^2+1)$
 $x^4 + x^3 + x^2 + x$

38) 4, -2i
 $(x-4)(x+2i)(x-2i)$
 $(x-4)(x^2+4)$

$$x^3 - 4x^2 + 4x - 16$$