Notes Section 4.2

Divide polynomials using long division and synthetic division
Use the Remainder Theorem to evaluate functions
Use the Factor Theorem to find zeros of a function

1-4 **Two polynomials P and D are given. Use either synthetic division or long division to divide P(x) by D(x), and express the quotient $\frac{P(x)}{D(x)}$ in the form:**

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

$x^3+4x^2-6x+1=(x-1)(x^2+5x-1)$

2. $P(x) = x^3 + 4x^2 - 6x + 1,\ D(x) = x - 1$

x^2+5x-1

$x-1\ \overline{)\,x^3+4x^2-6x+1}$

x^3-x^2

$5x^2-6x+1$

$5x^2-5x$

$-x+1$

$-x+1$

4. $P(x) = 2x^5 + 4x^4 - 4x^3 - x + 3,\ D(x) = x^2 - 2$

$(2x^5+4x^4-4x^3-x-3) =$
$(x^2-2)(2x^3+4x^2-8)$
$+(-x+13)$

$2x^3+4x^2+8$

$x^2-2\ \overline{)\,2x^5+4x^4-4x^3-x+3}$

$2x^5 \quad -4x^3$

$4x^4 \quad -x$

$4x^4-8x^2$

$8x^2-x+3$

$8x^2 \quad -16$

$-x+13$

5 - 14 **Find the quotient and remainder using long division.**

10. $\frac{3x^4-5x^3-20x-5}{x^2+x+3}$

$3x^2-8x-1$

$x^2+x+3\ \overline{)\,3x^4-5x^3 \quad -20x-5}$

$3x^4+3x^3+9x^2$

$-8x^3-9x^2-20x$

$-8x^3-8x^2-24x$

$-x^2+4x-5$

$-x^2-x-3$

$5x-2$

$(3x^2-8x-1);(5x-2)$

Synthetic Division:
Step 1: Write the coefficients in descending order (add zero's for missing terms)
Step 2: Write the constant r of the divisor $x-r$ to the left.
Step 3: Bring down the first coefficient
Step 4: Multiply the first coefficient by r. Write the product under the second coefficient.
Step 5: Add the product and the second coefficient
Step 6: Repeat steps 4 and 5 with remaining coefficients.

15 - 28 Find the quotient and remainder using synthetic division.

15. $\frac{x^2-5x+4}{x-3}$

$3\,|\ 1\ \ -5\ \ 4$
$\quad\ \ 3\ \ -6$
$\quad 1\ \ -2\ \ -2$

$(x-2);\ -2$

21. $\frac{x^3-8x+2}{x+3}$

$-3\,|\ 1\ \ 0\ \ -8\ \ 2$
$\quad\ \ -3\ \ 9\ \ -3$
$\quad 1\ \ -3\ \ 1\ \ -1$

$(x^2-3x+1);\ -1$

26. $\frac{6x^4+10x^3+5x^2+x+1}{x+\frac{2}{3}}$

$-\frac{2}{3}\,|\ 6\ \ 10\ \ 5\ \ 1\ \ 1$
$\quad\ \ -4\ \ -4\ \ -2/3\ \ -2/9$
$\quad 6\ \ 6\ \ 1\ \ 1/3\ \ 7/9$

$(6x^3+6x^2+x+\frac{1}{3});\ 7/9$

Remainder Theorem:
If the polynomial P(x) is divided by x – c, then the remainder is the value of P(c)

29 – 41 Use synthetic division and the Remainder Theorem to evaluate P(c)

34. $P(x) = 2x^3 - 21x^2 + 9x - 200, \quad c = 11$

$P(11) = 20$

$$\begin{array}{r|rrrr} 11 & 2 & -21 & 9 & -200 \\ & & 22 & 11 & 220 \\ \hline & 2 & 1 & 20 & 20 \end{array}$$

Factor Theorem:
***c* is a *zero* of *P* if and only if *x – c* is a factor of *P(x)*.**

43-46 Use the Factor Theorem to show that x – c is a factor of *P(x)* for the given value(s) of c.

44. $P(x) = x^3 + 2x^2 - 3x - 10, \quad c = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -3 & -10 \\ & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

yes $(x-2)$ is a factor

49-52 Find a polynomial of the specified degree that has the given zeros.

52. Degree 5; zeros $-2, -1, 0, 1, 2$

$(x+2)(x+1)(x)(x-1)(x-2)$

x^2-1

x^2-4

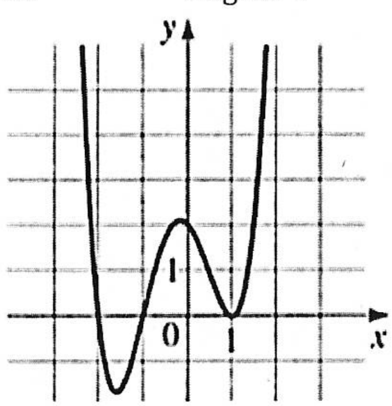
$x(x^2-1)(x^2-4)$

$x(x^4-5x^2+4)$

$\boxed{x^5-5x^3+4x}$

55-58 Find the polynomial of the specified degree whose graph is shown.

58. Degree 4



zero's are $-2, -1, 1, 1$

$(x+2)(x+1)(x-1)(x-1)$

x^2-1

x^2+x-2

$(x^2-1)(x^2+x-2) = x^4+x^3-2x^2$

$-x^2-x+2$

$\boxed{x^4+x^3-3x^2-x+2}$