

## Notes Section 7.5 Solving Rational Equations

I can solve a rational equation by finding common denominators.

I can solve a rational equation by factoring the denominators.

### Example 1:

Solve a rational equation by cross-multiplying.

$$\frac{20}{3x-5} = \frac{5}{x-2}$$

$$\begin{aligned} 20(x-2) &= 5(3x-5) \\ 20x - 40 &= 15x - 25 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Check:

$$\frac{20}{4} = \frac{5}{1}$$



✓ **Checkpoint** Solve the equation by cross multiplying.

$$1. \frac{3}{x+2} = \frac{6}{3x+8}$$

$$2. \frac{-6}{x+2} = \frac{-12}{x-1}$$

### Example 2:

Solve a rational equation with one solution.

$$\text{Solve: } \frac{8}{x} + \frac{11}{3} = \frac{-14}{x}$$

Find LCD:  $\underline{3x}$

$$3x\left(\frac{8}{x}\right) + 3x\left(\frac{11}{3}\right) = 3x\left(\frac{-14}{x}\right)$$

Check:

$$\frac{8}{-6} + \frac{11}{3} = \frac{-14}{-6} \quad \checkmark$$

$$24 + 11x = -42$$

$$11x = -66$$

$$x = -6$$

### Example 3:

Solve a rational equation with two solutions.

$$\text{Solve: } \frac{m+2}{m-1} + \frac{4}{m-5} = \frac{6}{m^2-6m+5}$$

Check

$$\boxed{-5} \quad \frac{-3}{-6} + \frac{4}{-10} = \frac{6}{60} \quad \checkmark$$

$$\boxed{4} \quad \frac{6}{3} + \frac{4}{-1} = \frac{6}{-3} \quad \checkmark$$

The LCD is  $(m-1)(m-5) = m^2-6m+5$

$$(m-1)(m-5)\left(\frac{m+2}{(m-1)}\right) + (m-1)(m-5)\left(\frac{4}{(m-5)}\right) = (m-1)(m-5)\left(\frac{6}{m^2-6m+5}\right)$$

$$(m-5)(m+2) + 4(m-1) = 6$$

$$m^2 - 3m - 10 + 4m - 4 = 6$$

$$m^2 + m - 14 = 6$$

$$m^2 + m - 20 = 6 \quad ; \quad \left( \begin{array}{l} (m+5)(m-4) = 0 \\ m = -5 \quad m = 4 \end{array} \right)$$

✔ **Checkpoint** Solve the equation by using the LCD.

$$3. \frac{11}{4} - \frac{3}{x} = \frac{5}{2x}$$

$$4. 1 + \frac{4}{x+2} = \frac{9}{x}$$

Example 4:

Check for extraneous solutions.

$$\frac{8x^2}{x^2-9} - \frac{4x}{x+3} = \frac{2}{x-3}$$

The LCD is  $(x+3)(x-3)$  or  $x^2-9$

$$\cancel{(x^2-9)} \left( \frac{8x^2}{\cancel{x^2-9}} \right) - \cancel{(x+3)} \cancel{(x-3)} \cdot \frac{4x}{\cancel{(x+3)}} = \cancel{(x+3)} \cancel{(x-3)} \cdot \frac{2}{\cancel{(x-3)}}$$

$$8x^2 - 4x(x-3) = 2(x+3) \rightarrow 4x^2 + 10x - 6 = 0$$

$$8x^2 - 4x^2 + 12x = 2x + 6$$

$$2(2x^2 + 5x - 3) = 0$$

$$2(2x-1)(x+3) = 0$$

✔ **Checkpoint** Solve the equation by using the LCD.  
Check for extraneous solutions.

$$\boxed{x = \frac{1}{2}} \quad | \quad \boxed{x = -3}$$

$$5. \frac{2m}{m-1} + \frac{m-5}{m^2-1} = 1$$