Section 7.4 Graphing of Rational Functions

Vocabulary:

Rational Function:

Function that has a variable in the denominator

Asymptotes:

line where the graph doesn't exist

Vertical Asymptote:

Vertical line that the function <u>can't</u> cross

Horizontal Asymptote:

horizontal line that the graph approaches (can be crossed)

Slant Asymptote:

when the numerator has a greater degree than the denominator. A slant line the graph can cross

Points of Discontinuity:

when an inputed variable creates a denominator of zero

Find the x- and y-intercepts of the function:

$s(x) = \frac{3x}{x-5}$

x-int: $3x = 0$
$x = 0$
$(0,0)$

y-int: $\frac{3(0)}{0-5}$
$y = 0$
$(0,0)$

$r(x) = \frac{2}{x^2+3x-4}$

x-int $2 \neq 0$
$\boxed{\text{no x-int}}$

y-int
$\frac{2}{(0^2+3(0)-4)}$
$\frac{2}{-4} = -\frac{1}{2}$
$\boxed{\text{y-int: } (0, -1/2)}$

Identify the Asymptotes:

$y = \frac{2x+5}{x-1}$

V.A: $x-1 = 0$
$\boxed{x = 1}$

H.A. (same degree) $\frac{2x}{x} = 2$
$\boxed{y = 2}$

$y = \frac{3x^2+4x-2}{x^3-x^2-6x}$

V.A. x^3-x^2-6x
$x(x^2-x-6)$
$x(x+2)(x-3)$

V.A. $x=0, x=-2, x=3$

H.A $n < d$
$\boxed{y = 0}$

$y = \frac{3x^3+4x-2}{x+7}$

V.A: $x+7=0$
$\boxed{x = -7}$

$\boxed{\text{H.A. none}}$ $n > d$
slant

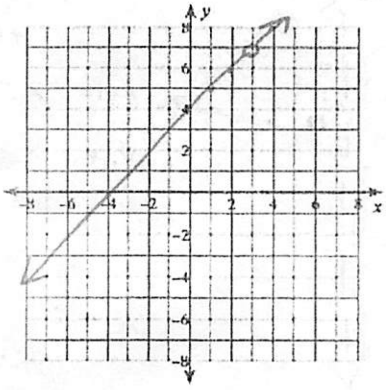
$y = \frac{x^2+4x+4}{x^2-2x-8}$

$y = \frac{x^2+4x-6}{x^3-3x^2-40x}$

Section 7.4 (Notes)

Graphing Rational Functions without a calculator:
Find the vertical and horizontal asymptotes and sketch a graph of the function:

$f(x) = \frac{x^2+x-12}{x-3} = \frac{(x+4)\cancel{(x-3)}}{\cancel{(x-3)}}$

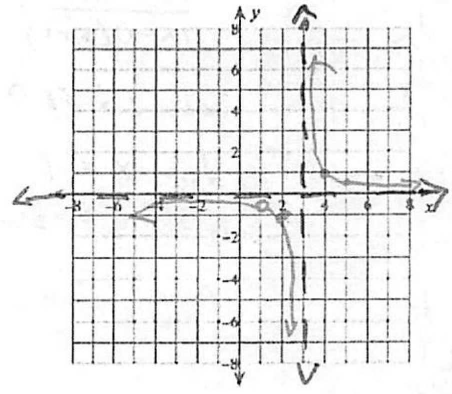


hole: $x=3$

V.A: none

H.A: none

$f(x) = \frac{x-1}{x^2-4x+3} = \frac{\cancel{x-1}}{(x-3)\cancel{(x-1)}} = \frac{1}{x-3}$



hole: $x=1$

V.A: $x=3$

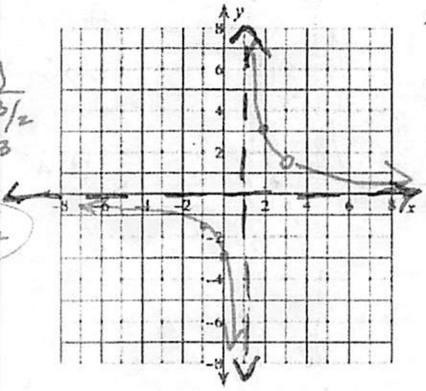
H.A: $y=0$

x	y
1	-1/2 → hole
2	-1
4	1
5	1/2

$f(x) = \frac{3x-9}{x^2-4x+3} = \frac{3\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \frac{3}{x-1}$

x	y
-1	-3/2
0	-3
2	3
3	3/2

hole

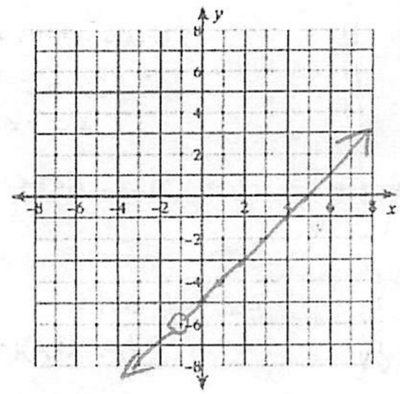


hole: $x=3$

V.A: $x=1$

H.A: $y=0$

$f(x) = \frac{x^2-4x-5}{x+1}$ $\frac{(x-5)\cancel{(x+1)}}{\cancel{(x+1)}}$

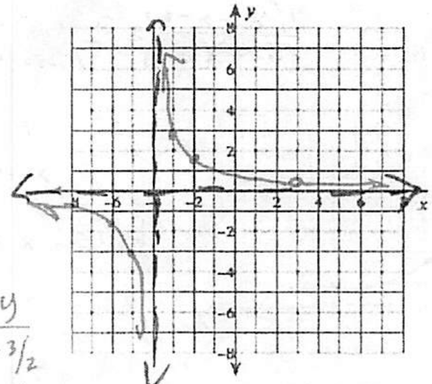


hole: $x=-1$

V.A: none

H.A: none

$f(x) = \frac{3x-9}{x^2+x-12} = \frac{3\cancel{(x-3)}}{\cancel{(x-3)}(x+4)} = \frac{3}{x+4}$



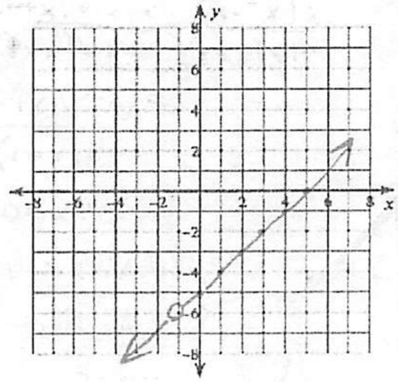
hole: $x=3$

V.A: $x=-4$

H.A: $y=0$

x	y
-6	-3/2
-5	-3
-3	3
-2	3/2

$f(x) = \frac{x^2-4x-5}{x+1}$ $\frac{(x-5)\cancel{(x+1)}}{\cancel{(x+1)}}$



hole: $x=-1$

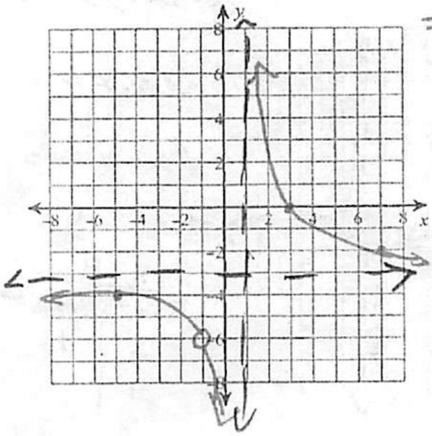
V.A: none

H.A: none

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each. Then sketch the graph using Desmos.

1) $f(x) = \dfrac{-3x^2 + 6x + 9}{x^2 - 1} = \dfrac{-3(x^2-2x-3)}{(x-1)(x+1)}$

$= \dfrac{-3(x-3)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$



hole: $x = 1$

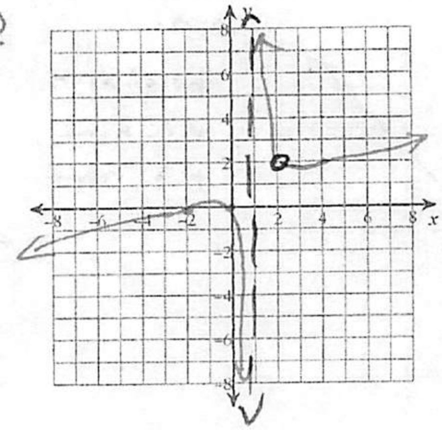
V.A: $x = 1$

H.A: $y = -3$

x-int: $(3,0)$

2) $f(x) = \dfrac{x^3 - 4x}{4x^2 - 12x + 8}$ $\dfrac{x(x^2-4)}{4(x^2-3x+2)} = \dfrac{x\cancel{(x-2)}(x+2)}{4\cancel{(x-2)}(x-1)}$

$= \dfrac{x(x+2)}{4(x-1)}$



hole: $x = 2$

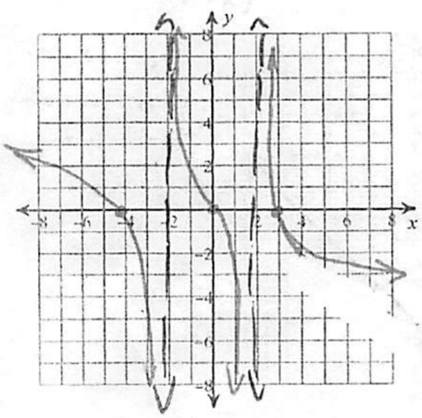
V.A: $x = 1$

H.A: none

Slant

x-int: $(0,0)$ $(0,-2)$

3) $f(x) = \dfrac{x^3 + x^2 - 12x}{-3x^2 + 12} = \dfrac{x(x^2+x-12)}{-3(x^2-4)} = \dfrac{x(x+4)(x-3)}{-3(x+2)(x-2)}$



Holes: none

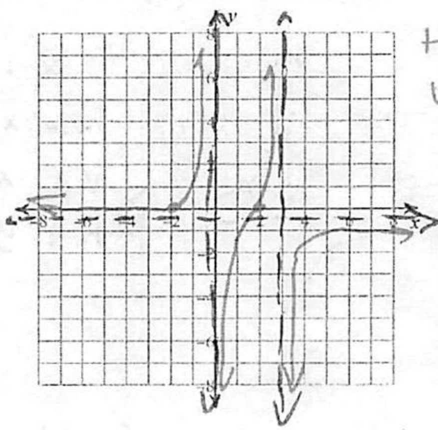
V.A: $x = 2$
$x = -2$

H.A: none

slant

X-int: $(0,0)$ $(3,0)$ $(-4,0)$

4) $f(x) = \dfrac{x^2 - 4}{-3x^2 + 9x} = \dfrac{(x+2)(x-2)}{-3x(x-3)}$



Holes: none

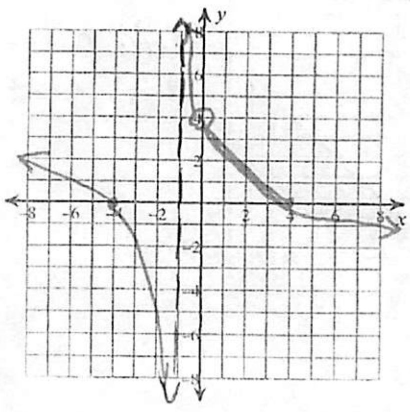
V.A: $x = 0$
$x = 3$

H.A: $y = -\frac{1}{3}$

x-int: $(-2,0)(2,0)$

5) $f(x) = \dfrac{x^3 - 16x}{-4x^2 - 4x}$ $\dfrac{x(x^2-16)}{-4x(x+1)} = \dfrac{\cancel{x}(x-4)(x+4)}{-4\cancel{x}(x+1)}$

$= \dfrac{(x-4)(x+4)}{-4(x+1)}$



hole: $x = 0$

V.A: $x = -1$

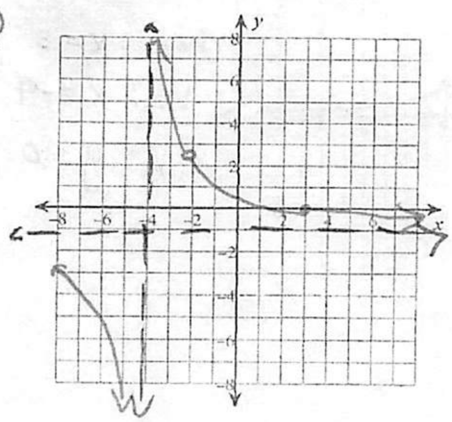
H.A: None

Slant

x-int: $(4,0)$ $(-4,0)$

6) $f(x) = \dfrac{-x^2 + x + 6}{x^2 + 6x + 8}$ $\dfrac{-1(x^2-x-6)}{(x+2)(x+4)} = \dfrac{-1(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x+4)}$

$= \dfrac{-(x-3)}{x+4}$



hole: $x = -2$

V.A: $x = -4$

H.A: $y = -1$

x-int: $(3,0)$