Notes

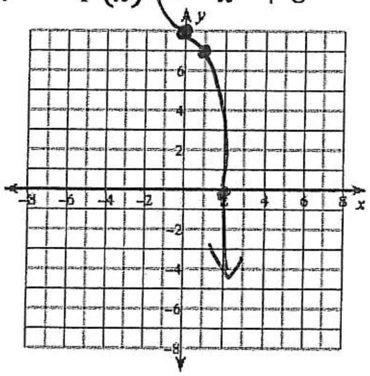
Notes Section 4.1

Graphing Polynomial Functions:

Using the End Behavior; Using the x-intercepts (zeros); Using the y-intercepts; Using test points

1-10 Sketch the graph of the function by transforming the graph of an appropriate function of the form $y = x^n$. Indicate all x- and y-intercepts on each graph.

2. $P(x) = -x^3 + 8$



up 8

flip on x-axis

End Behavior of Polynomials

		Degree	
		Even	**Odd**
Leading Coefficient	(+)	$x \to +\infty$ $f(x) \to +\infty$ $x \to -\infty$ $f(x) \to +\infty$	$x \to +\infty$ $f(x) \to +\infty$ $x \to -\infty$ $f(x) \to -\infty$
	(–)	$x \to +\infty$ $f(x) \to -\infty$ $x \to -\infty$ $f(x) \to -\infty$	$x \to +\infty$ $f(x) \to -\infty$ $x \to -\infty$ $f(x) \to +\infty$

Real Zeros of Polynomials

If P is a polynomial and c is a real number, then the following are equivalent.

1. c is a zero
2. $(x-c)$ is a factor
3. $x=c$ is a solution of the equation $P(x)=0$
4. $(c,0)$ is an x-intercept of the graph of $P(x)$.

Guidelines for Graphing Polynomial Functions:

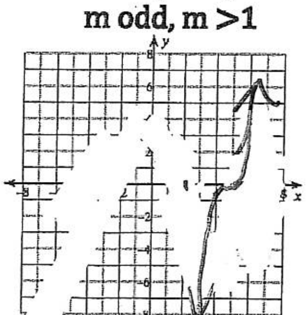
1. Zeros Factor to find all the zeros
2. Test Points make a table of values around the x-int.
3. End Behavior Determine the end behavior
4. Graph make a smooth curve for the graph

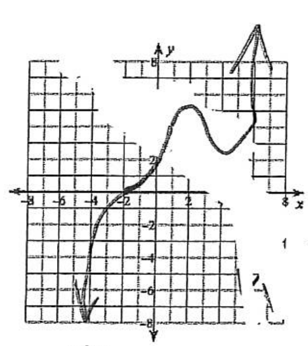
Shape of the Graph Near a Zero of Multiplicity m

Suppose that c is a zero of $P(x)$ of multiplicity m. Then the shape of the graph of $P(x)$ near c is as follows:

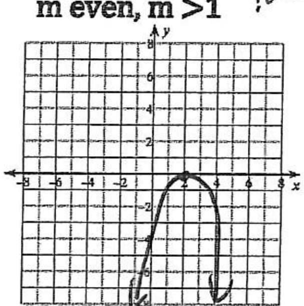
Multiplicity of c

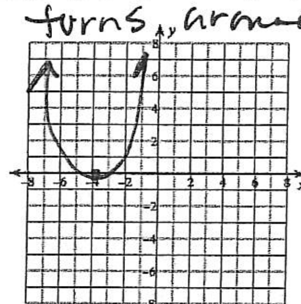
m odd, m >1





m even, m >1 touches the intercept and turns around





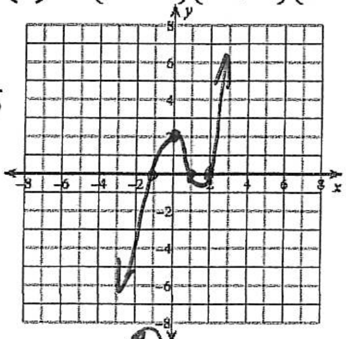
odd behaves like a cubic when m > 1 and odd

Notes con't.

17-28 Sketch the graph of the polynomial function. Make sure your graph shows all intercepts and exhibits the proper end behavior.

$(1.5-1)(1.5+1)(1.5-2)$

18. $P(x) = (x-1)(x+1)(x-2)$



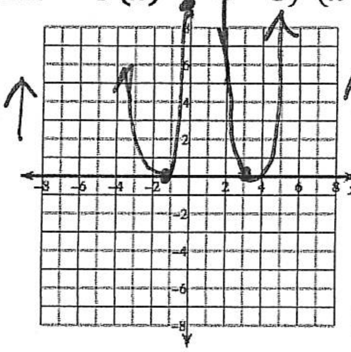
1.5 | -.625

end behavior
D: 0
L.C. (+)

Zeros
1, -1, 2
y-int: (0, 2)

$(-2)^2(2)^2$

28. $P(x) = (x-3)^2(x+1)^2$



End behav.
D: even
LC: (+)

Zeros
3 mult 2
-1 mult 2

y-int (0, 9)

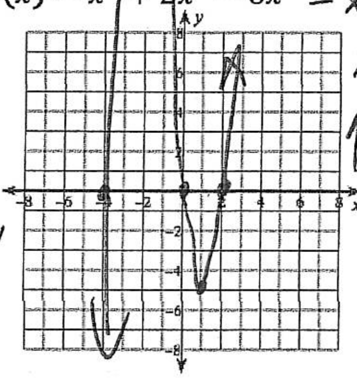
x	y
1	16

29-42 Factor the polynomial and use the factored form to find the zeros. Then sketch the graph.

-2.5 | 1.17

30. $P(x) = x^3 + 2x^2 - 8x = x(x^2+2x-8)$

$x(x+4)(x-2)$



End Behav.
D: 0
LC: +

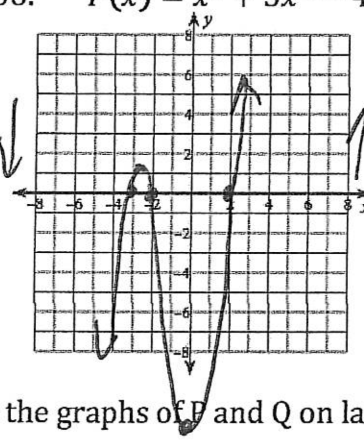
zeros
0, -4, 2

y-int
(0, 0)

x	y
1	-5
-2	16

36. $P(x) = x^3 + 3x^2 - 4x - 12$

$x^2(x+3)-4(x+3)$
$(x^2-4)(x+3)$
$(x+2)(x-2)(x+3)$



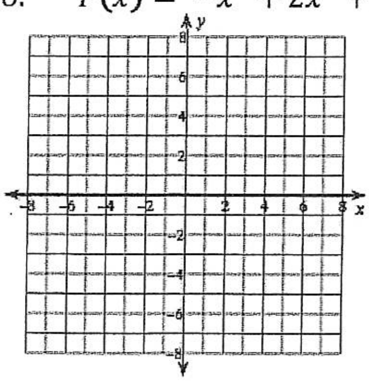
zeros
$\pm 2, -3,$

y-int
(0, -12)

End Behavior
D: odd LC: +

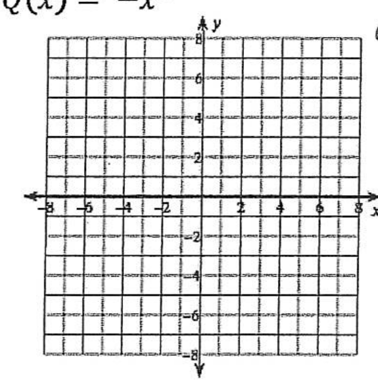
43-48 Determine the end behavior of P. Compare the graphs of P and Q on large and small viewing rectangles, as in example 3(b)

46. $P(x) = -x^5 + 2x^2 + x;$



Use Desmos, or graphing Calculator

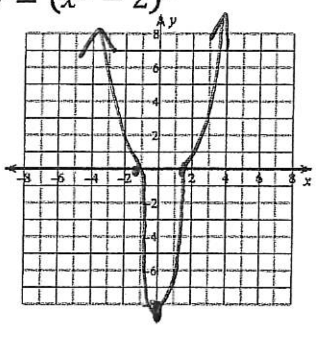
$Q(x) = -x^5$



When you zoom out the graphs get closer & closer

61-70 Graph the polynomial and determine how many local maxima and minima it has.

68. $y = (x^2-2)^3$



Use graphing Calculator

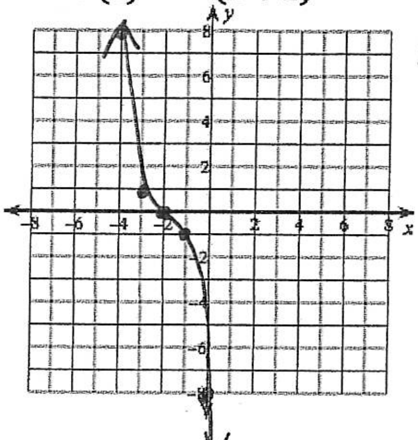
$x^2 - 2 = 0$

$x = \pm\sqrt{2}$

(Homework)

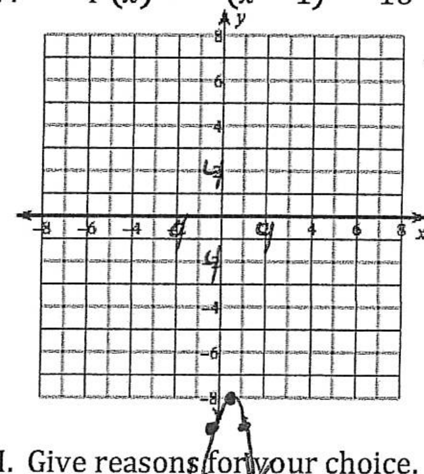
1-10 Sketch the graph of the function by transforming the graph of an appropriate function of the form $y = x^n$. Indicate all x- and y-intercepts on each graph.

3. $P(x) = -(x+2)^3$



flip on x-axis
left 2

7. $P(x) = -(x-1)^4 - 16$



flip on x-axis
down 16
right 1

11-16 Match the polynomial function with one of the graphs I-VI. Give reasons for your choice.

11. $P(x) = x(x^2-4)$ III

odd degree (+) Leading Coeficcent

13. $P(x) = -x^5 + 5x^3 - 4x$

$-x(x^4 - 5x^2 + 4)$

$-x(x^2-1)(x^2-4)$

zeros at $0, \pm 1, \pm 2$

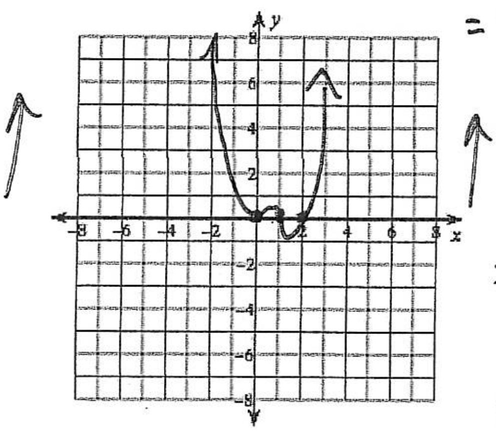
odd degree (–) leading coefficient $\boxed{V}$

29-42 Factor the polynomial and use the factored form to find the zeros. Then sketch the graph.

33. $P(x) = x^4 - 3x^3 + 2x^2$

$x^2(x^2 - 3x + 2)$

$= x^2(x-2)(x-1)$

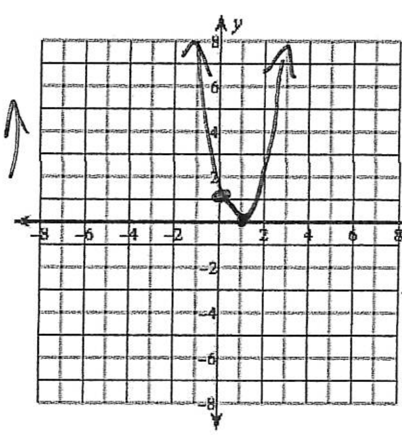


Zeros
0 mult 2
1, 2

x	y
$\frac{1}{2}$	.1875 or $3/16$
$\frac{3}{2}$	–.84 or $-\frac{27}{32}$

y-int (0,0)

41. $P(x) = x^6 - 2x^3 + 1$



$(x^3-1)(x^3-1)$

$(x-1)(x^2+x+1)$

$(x-1)(x^2+x+1)$

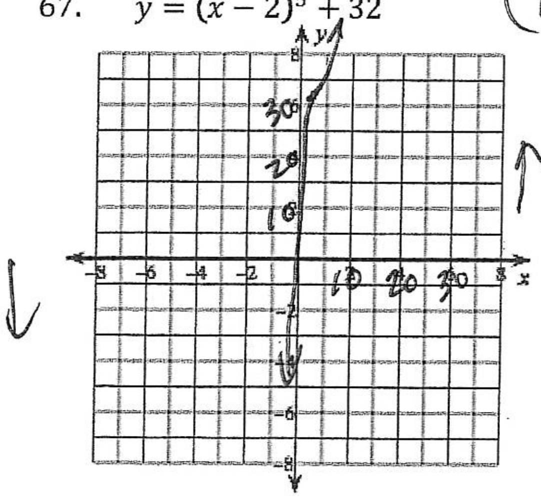
$(x-1)^2(x^2+x+1)^2$

Zeros
1 mult 2
y-int (0,1)

61-70 Graph the polynomial and determine how many local maxima and minima it has.

67. $y = (x-2)^5 + 32$

(Use Desmos or graphing calculator)



No max or min.