Conic Sections

Circles:

- Identify the center and radius from the standard equation of a circle
- Graph equation of a circle
- Write a general form of an equation from the standard form.
- Write a standard form of an equation from the general form.

What are conic sections?

There are four types of conic sections:

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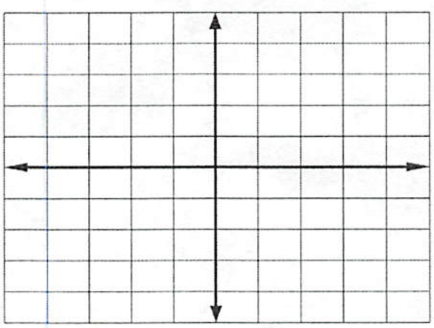
-
-
-

Standard Equations of a Circle:

$(x+2)^2 + (y+1)^2 = 9$



Find the center and radius of each equation of a circle:

$x^2 + (y-2)^2 = 16$

$(x-4)^2 + (y+5)^2 = 20$

Sometimes, the equations of circles are not in standard form:

Example:

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

We use the technique **"completing the square"** to rewrite the equation in standard form:

Step 1: Isolate the constant term.

Step 2: Put parenthesis around your terms with the same variables.

Step 3: Make sure each squared term is 1. If it isn't, factor that value out of each expression.

Step 4: Complete the square by dividing by "b" in each expression by 2, then square it and add it to each expression.

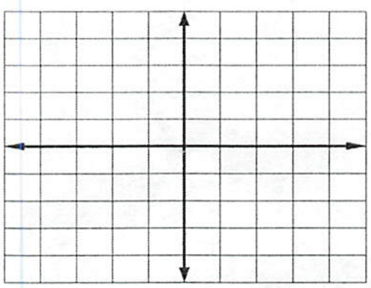
Step 5: Simplify both sides of the equation by factoring the quantities and adding the constants.

You now have the equation of a circle written in standard form.

The center and radius are:

Find the center and radius of the following circle and sketch its graph:

$$x^2 + 2x + y^2 - 4y + 4 = 0$$



Complete the square when "a" is not 1 (Use the 5 Steps from above)

$$9x^2 + 5y^2 + 72x - 60y + 288 = 0$$

$$2x^2 + y^2 + 12x - 4y - 3 = 0$$

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- Identify the center and radius from the standard equation of a circle
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What are conic sections?

formed by the intersection of a cone and a plane

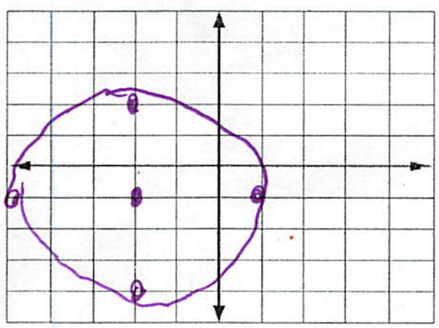
There are four types of conic sections:

- Ellipses
- Parabolas
- Circles
- Hyperbolas

Standard Equations of a Circle:

$(x-h)^2+(y-k)^2=r^2$ (h,k) Center
r: radius

$(x+2)^2+(y+1)^2=9$



(-2,-1) Center
$r=3$

Find the center and radius of each equation of a circle:

$x^2+(y-2)^2=16$

C(0,2) $r=4$

$(x-4)^2+(y+5)^2=20$

C(4,-5) $r=\sqrt{20}=2\sqrt{5}$

Sometimes, the equations of circles are not in standard form:

Example:

$x^2 + y^2 + 4x - 6y - 3 = 0$

We use the technique **"completing the square"** to rewrite the equation in standard form:

Step 1: Isolate the constant term. $x^2 + y^2 + 4x - 6y = 3$

Step 2: Put parenthesis around your terms with the same variables.

$(x^2 + 4x) + (y^2 - 6y) = 3$

Step 3: Make sure each squared term is 1. If it isn't, factor that value out of each expression.

Step 4: Complete the square by dividing by "b" in each expression by 2, then square it and add it to each expression. $(x^2 + 4x + 4) + (y^2 - 6y + 9) = 3 + 4 + 9$

Step 5: Simplify both sides of the equation by factoring the quantities and adding the constants.

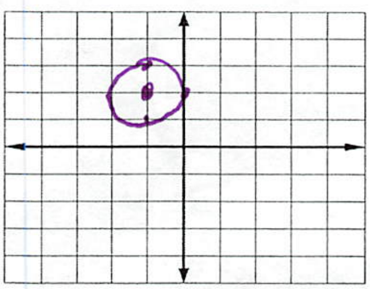
$(x+2)^2 + (y-3)^2 = 16$

You now have the equation of a circle written in standard form.

The center and radius are: $C(-2, 3)$ $r = 4$

Find the center and radius of the following circle and sketch its graph:

$x^2 + 2x + y^2 - 4y + 4 = 0$



$(x^2 + 2x) + (y^2 - 4y) = -4$

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = -4 + 1 + 4$

$(x+1)^2 + (y-2)^2 = 1$

$C(-1, 2)$ $r = 1$

Complete the square when "a" is not 1 (Use the 5 Steps from above)

$9x^2 + 5y^2 + 72x - 60y + 288 = 0$

$(9x^2 + 72x) + (5y^2 - 60y) = -288$

$9(x^2 + 8x + 16) + 5(y^2 - 12y + 36) = -288 + 9(16) + 5(36)$

$9(x+4)^2 + 5(y-6)^2 = 36$

$2x^2 + y^2 + 12x - 4y - 3 = 0$ $(2x^2 + 12x) + (y^2 - 4y) = 3$

$2(x^2 + 6x) + (y^2 - 4y) = 3$

$2(x^2 + 6x + 9) + (y^2 - 4y) = 3 + 2(9)$

$2(x+3)^2 + (y-2)^2 = 21$

Conic Sections

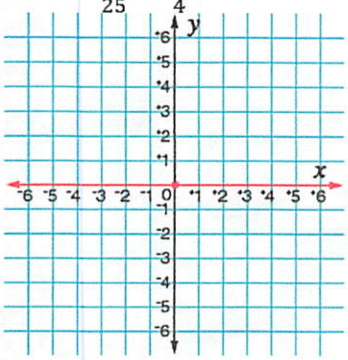
Ellipse:

- I know the differences and similarities of an equation of a circle and an ellipse
- I can write the general form of an equation to standard form of an ellipse
- I can graph an equation of an ellipse and identify vertices and foci.
- I can find the eccentricity and the area of an ellipse
- I can write a general form of an equation of an ellipse from the standard form.
- I can write a standard form of an equation of an ellipse from the general form.

Formula for an ellipse:

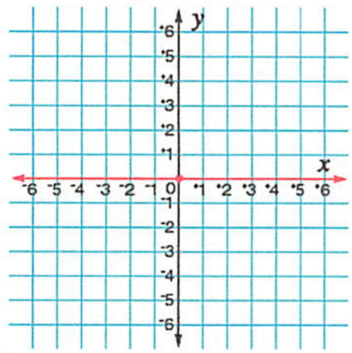
Graph:

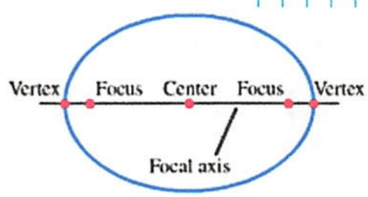
$$\frac{x^2}{25}+\frac{y^2}{4}=1$$



Sometime equations doesn't come equal to 1 ☺

$$25(x+2)^2+9(y-1)^2=225$$





Graph the equations.

Make sure you include the direction of the ellipse, center, all 4 vertices, (major and minor), foci, eccentricity and area.

1. $\frac{x^2}{16}+\frac{y^2}{49}=1$

direction:__________

center: __________

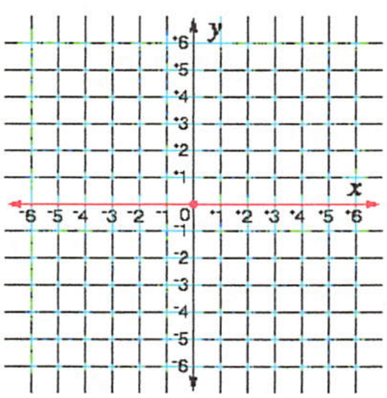
major vertices __________

minor vertices __________

focus __________

eccentricity __________

area __________



2. $\frac{(x-4)^2}{9}+\frac{(y+5)^2}{36}=1$

direction: __________

center: __________

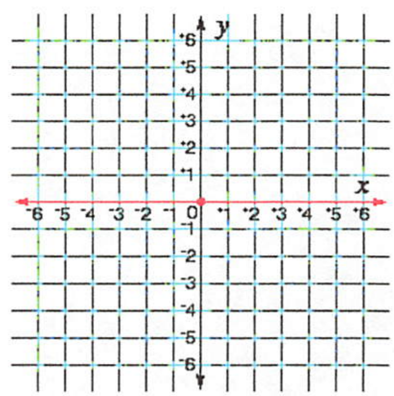
major vertices __________

minor vertices __________

focus __________

eccentricity __________

area __________



Write each of the following equations of an ellipse in general form:

$$\frac{(x+3)^2}{25}+\frac{(y-4)^2}{4}=1$$

Write each of the following equations of an ellipse in standard form:

$$7x^2+10y^2-42x+60y+83=0$$

Using graph paper, graph each of the following. Make sure you identify the direction of the ellipse, center, vertices (all 4 major and minor), foci, eccentricity, and the area.

3. $\frac{(x-3)^2}{1}+\frac{(y+1)^2}{24}=1$

direction:__________

center: __________

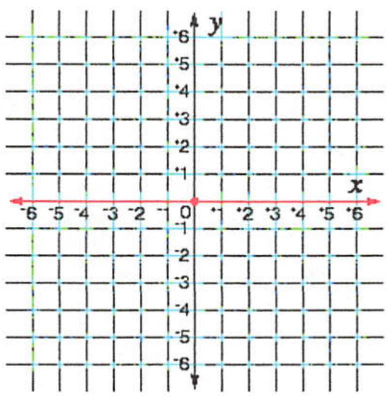
major vertices __________

minor vertices __________

focus __________

eccentricity ________

area ________



4. $10x^2+9y^2-20x+54y+1=0$

direction: __________

center: __________

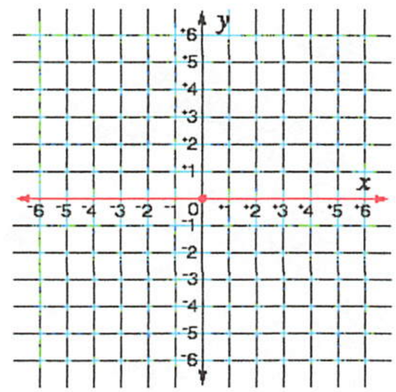
major vertices __________

minor vertices __________

focus __________

eccentricity ________

area ________



Find an equation for the ellipse that satisfies the given condition:

Foci $(\pm4, 0)$, Vertices $(\pm5, 0)$

Length of major axis 4, length of the minor axis 2, Center is (0, 0), Foci is on the y-axis

Foci $(0, \pm2)$, length of the minor axis is 6.

Endpoints of the major axis $(\pm10, 0)$, distance between foci 6

Conic Sections

Ellipse:

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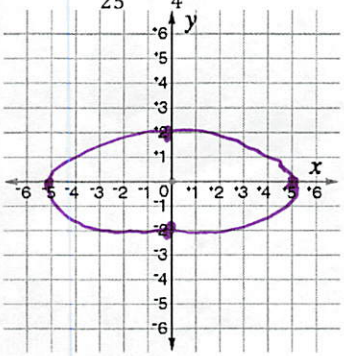
- I can write a standard form of an equation of an ellipse from the general form.

Formula for an ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

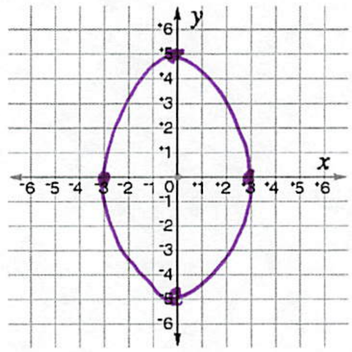
Graph:

$$\frac{x^2}{25}+\frac{y^2}{4}=1$$

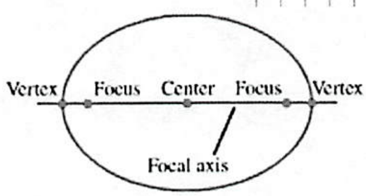


Sometime equations doesn't come equal to 1 ☺

$$25(x+2)^2+9(y-1)^2=225$$



$\frac{(x+2)^2}{9}+\frac{(y-1)^2}{25}=1$



Graph the equations.

Make sure you include the direction of the ellipse, center, all 4 vertices, (major and minor), foci, eccentricity and area.

1. $\frac{x^2}{16}+\frac{y^2}{49}=1$

direction: Vert.

center: (0,0)

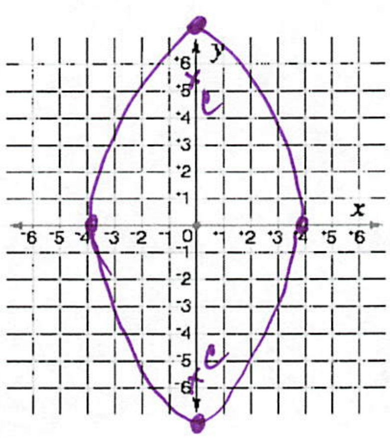
a major vertices (0,7) (0,-7)

b minor vertices (4,0) (-4,0)

focus (0, ±√33)

eccentricity √33/7

area 28π



$c^2=a^2-b^2$

$c^2=49-16$

$c=\sqrt{33}$

$e=\frac{c}{a}$

2. $\frac{(x-4)^2}{9}+\frac{(y+5)^2}{36}=1$

direction: Vert.

center: (4,-5)

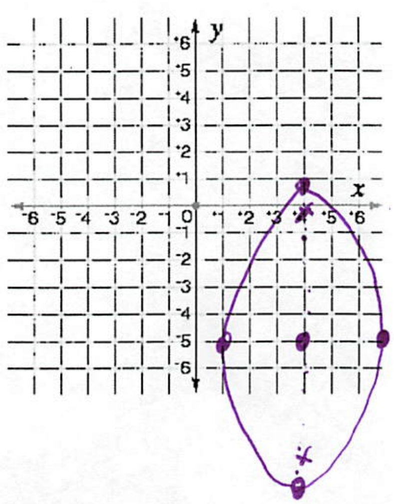
major vertices (4,1) (4,-11)

minor vertices (7,-5) (1,-5)

focus (4,0),(4,-10)

eccentricity 5/6

area 18π



$c^2=36-9$

$c=5$

Write each of the following equations of an ellipse in general form:

$$\frac{(x+3)^2}{25} + \frac{(y-4)^2}{4} = 1$$

$4(x+3)^2 + 25(y-4)^2 = 1$

$4(x^2+6x+9) + 25(y^2-8y+16) = 1$

$4x^2 + 24x + 36 + 25y^2 - 200y + 400 = 1$

$4x^2 + 25y^2 + 24x - 200y + 336 = 0$

Write each of the following equations of an ellipse in standard form:

$$7x^2 + 10y^2 - 42x + 60y + 83 = 0$$

$7x^2 - 42x + 10y^2 + 60y = -83$

$7(x^2-6x+9) + 10(y^2+6y+9) = -83 + 7(9) + 10(9)$

$7(x-3)^2 + 10(y+3)^2 = 70$

$\frac{(x-3)^2}{10} + \frac{(y+3)^2}{7} = 1$

Using graph paper, graph each of the following. Make sure you identify the direction of the ellipse, center, vertices (all 4 major and minor), foci, eccentricity, and the area.

3. $\frac{(x-3)^2}{1} + \frac{(y+1)^2}{24} = 1$

direction: Vert.

center: __________

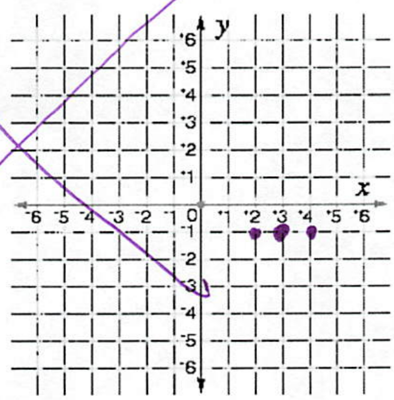
major vertices __________

minor vertices __________

focus __________

eccentricity __________

area __________



4. $10x^2 + 9y^2 - 20x + 54y + 1 = 0$

direction: __________

center: __________

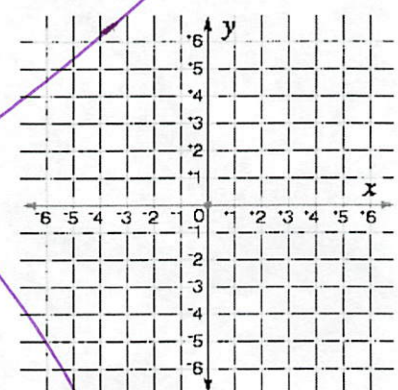
major vertices __________

minor vertices __________

focus __________

eccentricity __________

area __________



Find an equation for the ellipse that satisfies the given condition:

Foci $(\pm 4, 0)$, Vertices $(\pm 5, 0)$

$a = 5 \quad c = 4$

$c^2 = a^2 - b^2$

$16 = 25 - b^2$

$b = 3$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

Length of major axis 4, length of the minor axis 2, Center is (0, 0), Foci is on the y-axis

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$2a = 4$

$a = 2$

$2b = 2$

$b = 1$

$\frac{x^2}{1} + \frac{y^2}{4} = 1$

Foci $(0, \pm 2)$, length of the minor axis is 6.

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$c = 2$

$2b = 6 \quad b = 3$

$4 = a^2 - 9$

$a^2 = 13$

$\frac{x^2}{9} + \frac{y^2}{13} = 1$

Endpoints of the major axis $(\pm 10, 0)$, distance between foci 6

$a = 10$

$2c = 6$

$c = 3$

$3^2 = 10^2 - b^2$

$b^2 = 91$

$\frac{x^2}{100} + \frac{y^2}{91} = 1$

Definition for Ellipse:

Area of Ellipse:

Ellipse with Center (0,0)

Equation		
Vertices (Major)		
Co-Vertices (Minor)		
Major Axis		
Minor Axis		
Foci		
Eccentricity		
Graph		

Definition of Ellipse: The sum of the distances from the foci to each point on the ellipse is a constant

Area of Ellipse: $A = \pi ab$

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a>b>0$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ $a>b>0$
Vertices (Major)	$(\pm a, 0)$	$(0, \pm a)$
Co-Vertices (Minor)	$(0, \pm b)$	$(\pm b, 0)$
Major Axis	*Length of* $2a$ (H) $y=k$	*Length of* $2a$ (V) $x=h$
Minor Axis	*Length of* $2b$ V	*Length of* $2b$ H
Foci	$(\pm c, 0)$ $c^2=a^2-b^2$	$(0, \pm c)$ $c^2=a^2-b^2$
Eccentricity	$e=\frac{c}{a}$	$e=\frac{c}{a}$
Graph		

Ellipse with Center (h,k)

Equation		
Vertices (Major)		
Vertices (Major)		
Major Axis		
Minor Axis		
Foci		
Eccentricity		
Graph		

$A = \pi a b$

Ellipse with Center (h,k)

Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $a > b > 0$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ $a > b > 0$
Vertices (Major)	$(h \pm a, k)$	$(h, k \pm a)$
Co-Vertices (Minor)	$(h, k \pm b)$	$(h \pm b, k)$
Major Axis	$Length\ of\ 2a$ $(H)\ y = k$	$Length\ of\ 2a$ $(V)\ x = h$
Minor Axis	$Length\ of\ 2b$ V	$Length\ of\ 2b$ H
Foci	$(h \pm c, k)$ $c^2 = a^2 - b^2$	$(h, k \pm c)$ $c^2 = a^2 - b^2$
Eccentricity	$e = \frac{c}{a}$	$e = \frac{c}{a}$
Graph		