Notes 7.1 Solving Systems of Linear Equations
With Matrices

Solve systems of equations using:

- What are the dimensions of a matrix?
- Change linear systems into augmented matrices.
- Solve a system using Gaussian Elimination.
-

7–14 ▪ A matrix is given.

(a) Determine whether the matrix is in row-echelon form.

(b) Determine whether the matrix is in reduced row-echelon form.

(c) Write the system of equations for which the given matrix is the augmented matrix.

8. $\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 5 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 0 & -7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

15–24 ▪ The system of linear equations has a unique solution. Find the solution using Gaussian elimination or Gauss-Jordan elimination.

17.
$$\begin{cases} x + y + z = 2 \\ 2x - 3y + 2z = 4 \\ 4x + y - 3z = 1 \end{cases}$$

18.
$$\begin{cases} x + y + z = 4 \\ -x + 2y + 3z = 17 \\ 2x - y = -7 \end{cases}$$

27. $$\begin{cases} 2x - 3y - 9z = -5 \\ x \quad\quad + 3z = 2 \\ -3x + y - 4z = -3 \end{cases}$$

29. $$\begin{cases} x - y + 3z = 3 \\ 4x - 8y + 32z = 24 \\ 2x - 3y + 11z = 4 \end{cases}$$

35. $$\begin{cases} 4x - 3y + z = -8 \\ -2x + y - 3z = -4 \\ x - y + 2z = 3 \end{cases}$$

42. $$\begin{cases} x - 3y + 2z + w = -2 \\ x - 2y \quad\quad - 2w = -10 \\ \quad\quad\quad z + 5w = 15 \\ 3x \quad\quad + 2z + w = -3 \end{cases}$$

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7–14 ▪ A matrix is given.

(a) Determine whether the matrix is in row-echelon form.

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(c) Write the system of equations for which the given matrix is the augmented matrix.

8. $\begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 5 \end{bmatrix}$ yes no $x+3y=-3$, $y=5$

10. $\begin{bmatrix} 1 & 0 & -7 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ yes no $x-7z=0$, $y+3z=0$, $0=1$

15–24 ▪ The system of linear equations has a unique solution. Find the solution using Gaussian elimination or Gauss-Jordan elimination.

17. $\begin{cases} x+y+z=2 \\ 2x-3y+2z=4 \\ 4x+y-3z=1 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -3 & 1 \end{bmatrix} \overset{(-2)}{=} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 4 & +1 & -3 & 1 \end{bmatrix} \overset{(-4)}{=} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -7 & -7 \end{bmatrix} 3$

$= \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -7 & -7 \end{bmatrix} (-1) = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} (-1) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} =$ **(1, 0, 1)**

18. $\begin{cases} x+y+z=4 \\ -x+2y+3z=17 \\ 2x-y=-7 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & 4 \\ -1 & 2 & 3 & 17 \\ 2 & -1 & 0 & -7 \end{bmatrix} \overset{(-2)}{=} \begin{bmatrix} 1 & 1 & 1 & 4 \\ -1 & 2 & 3 & 17 \\ 0 & -3 & -2 & -15 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & -3 & -2 & -15 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 1 & 3 \end{bmatrix} =$ $z=3$, $3y+4z=21$, $3y+12=21$, $3y=9$, $y=3$

$x+y+z=4$, $x+3+3=4$, $x=-2$

(−2, 3, 3)

27. $\begin{cases} 2x - 3y - 9z = -5 \\ x + 3z = 2 \\ -3x + y - 4z = -3 \end{cases}$

29. $\begin{cases} x - y + 3z = 3 \\ 4x - 8y + 32z = 24 \\ 2x - 3y + 11z = 4 \end{cases}$

$\begin{bmatrix} 1 & -1 & 3 & 3 \\ 4 & -8 & 32 & 24 \\ 2 & -3 & 11 & 4 \end{bmatrix} \begin{matrix} \\ \\ (-2) \end{matrix} \cdot = \begin{bmatrix} 1 & -1 & 3 & 3 \\ 4 & -8 & 32 & 24 \\ 0 & -1 & 5 & -2 \end{bmatrix} \begin{matrix} \\ \\ (-4) \end{matrix}$

$= \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & -4 & 20 & 12 \\ 0 & -1 & 5 & -2 \end{bmatrix} \div 4 = \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -5 & -3 \\ 0 & -1 & 5 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -5 & -3 \\ 0 & 0 & 0 & -5 \end{bmatrix}$

$0 \neq -5$

no solution

35. $\begin{cases} 4x - 3y + z = -8 \\ -2x + y - 3z = -4 \\ x - y + 2z = 3 \end{cases}$

$(-2, 1, 0, 3)$

42. $\begin{cases} x - 3y + 2z + w = -2 \\ x - 2y - 2w = -10 \\ z + 5w = 15 \\ 3x + 2z + w = -3 \end{cases}$

$\begin{matrix} \\ (-3) \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 1 & -2 & 0 & -2 & -10 \\ 0 & 0 & 1 & 5 & 15 \\ 3 & 0 & 2 & 1 & -3 \end{bmatrix} \begin{matrix} \\ (-1) \\ \\ \\ \end{matrix} = \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 9 & -4 & -2 & 3 \end{bmatrix} \begin{matrix} \\ \\ \\ (-9) \end{matrix}$

$= \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 0 & 14 & 25 & 75 \end{bmatrix} \begin{matrix} \\ \\ \\ (-14) \end{matrix} = \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 0 & 0 & -45 & -135 \end{bmatrix} =$

$-45z = -135$

$z = 3$

$y + 5z = 15$

$y + 15 = 15$

$y = 0$

$x - 2y - 3z = 8$

$x - 9 = -8$

$x = 1$

$w - 3x + 2y + z = -2$

$w - 3 + 3 = -2$

$w = -2$