**The Binomial Theorem:**

- Expand a Binomial Using Pascal's Triangle
- Calculate Binomial Coefficients
- Expand a Binomial Using the Binomial Theorem
- Factor a Binomial Using the Binomial Theorem

1-12 Use Pascal's triangle to expand the expression.

2. $(2x+1)^4$

12. $\left(2+\frac{x}{2}\right)^5$

***n* factorial:**

$$\boldsymbol{n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n}$$

We also define 0! as follows:

$$0! = 1$$

The Binomial Coefficient:
Let $n$ and $r$ be nonnegative integers with $r \leq n$.

The **binomial coefficient** is denoted by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

13-20 Evaluate the expression.

16. $\binom{10}{5}$

18. $\binom{5}{2}\binom{5}{3}$

20. $\binom{5}{0} - \binom{5}{1} + \binom{5}{2} - \binom{5}{3} + \binom{5}{4} - \binom{5}{5}$

The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$\binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

21-24 Use the Binomial Theorem to expand the expression.

22. $(1 - x)^5$

24. $(2A + B^2)^4$

26. Find the first four terms in the expansion of $\left(x^{1/2} + 1\right)^{30}$

28. Find the first three terms in the expansion of $\left(x + \frac{1}{x}\right)^{40}$

32. Find the 28th term in the expansion of $(A - B)^{30}$

36. Find the term containing $y^3$ in the expansion of $\left(\sqrt{2} + y\right)^{12}$

39-42 Factor using the binomial theorem

42. $x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$

**The Binomial Theorem:**

- Expand a Binomial Using Pascal's Triangle
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1-12 Use Pascal's triangle to expand the expression.

2. $(2x+1)^4$

$1 \quad 4 \quad 6 \quad 4 \quad 1$

$(2x)^4 \quad (2x)^3 \quad (2x)^2 \quad (2x)^1 \quad (2x)^0$

$1^0 \quad 1^1 \quad 1^2 \quad 1^3 \quad 1^4$

$\boxed{16x^4 + 32x^3 + 24x^2 + 8x + 1}$

12. $\left(2+\frac{x}{2}\right)^5$

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$(2)^5 \quad (2)^4 \quad (2)^3 \quad (2)^2 \quad (2)^1 \quad (2)^0$

$\left(\frac{x}{2}\right)^0 \quad \left(\frac{x}{2}\right)^1 \quad \left(\frac{x}{2}\right)^2 \quad \left(\frac{x}{2}\right)^3 \quad \left(\frac{x}{2}\right)^4 \quad \left(\frac{x}{2}\right)^5$

$\boxed{= 32 + 40x + 20x^2 + 5x^3 + \frac{5}{8}x^4 + \frac{x^5}{32}}$

***n* factorial:**

$$\boldsymbol{n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n}$$

We also define 0! as follows:

$$0! = 1$$

The Binomial Coefficient:

Let $n$ and $r$ be nonnegative integers with $r \le n$.

The **binomial coefficient** is denoted by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

13-20 Evaluate the expression.

16. $\binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 252$

18. $\binom{5}{2}\binom{5}{3} = \frac{5!}{2!(5-2)!} = \frac{5 \cdot 4 \cdot \cancel{3!}}{2 \cdot 1 \cdot \cancel{3!}} = 10$ $\qquad 10 \cdot 10 \boxed{= 100}$

20. $\binom{5}{0} - \binom{5}{1} + \binom{5}{2} - \binom{5}{3} + \binom{5}{4} - \binom{5}{5}$

$1 - 5 + 10 - 10 + 5 - 1$

$\boxed{= 0}$

$\frac{5!}{0!(5-0)!} = 1 \quad \frac{5!}{1!(5-1)!} = \frac{5 \cdot \cancel{4!}}{\cancel{4!}} = 5 \quad \frac{5!}{2!(5-2)!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 (3!)} = 10$

$\frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3!}{3! \, 2 \cdot 1} = 10 \quad \frac{5!}{4!(5-4)!} = \frac{5 \cdot 4!}{4! \cdot 1} = 5 \quad \frac{5!}{5!0!} = 1$

The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$\binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

21-24 Use the Binomial Theorem to expand the expression.

22. $(1-x)^5$

$\binom{5}{0}$ $\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$

$1$ $5$ $10$ $10$ $5$ $1$

$1^5$ $1^4$ $1^3$ $1^2$ $1^1$ $1^0$

$(-x)^0$ $(-x)^1$ $(-x)^2$ $(-x)^3$ $(-x)^4$ $(-x)^5$

$\boxed{= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5}$

24. $(2A+B^2)^4$

$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

$1$ $4$ $6$ $4$ $1$

$(2A)^4$ $(2A)^3$ $(2A)^2$ $(2A)^1$ $(2A)^0$

$(B^2)^0$ $(B^2)^1$ $(B^2)^2$ $(B^2)^3$ $(B^2)^4$

$16A^4 + 32A^3B^2 + 24A^2B^4 + 8AB^6 + B^8$

26. Find the first four terms in the expansion of $\left(x^{1/2}+1\right)^{30}$

$\binom{30}{0}$ $\binom{30}{1}$ $\binom{30}{2}$ $\binom{30}{3}$

$1$ $30$ $435$ $4060$

$(x^{1/2})^{30}$ $(x^{1/2})^{29}$ $(x^{1/2})^{28}$ $(x^{1/2})^{27}$ $= x^{15} + 30x^{\frac{29}{2}} + 435x^{14} + 4060x^{\frac{27}{2}}$

$1^0$ $1^1$ $1^2$ $1^3$

28. Find the first three terms in the expansion of $\left(x+\frac{1}{x}\right)^{40}$

$\binom{40}{0}$ $\binom{40}{1}$ $\binom{40}{2}$

$1$ $40$ $780$

$x^{40}$ $x^{39}$ $x^{38}$ $= x^{40} + 40x^{38} + 780x^{36}$

$(\frac{1}{x})^0$ $(\frac{1}{x})^1$ $(\frac{1}{x})^2$

32. Find the 28th term in the expansion of $(A-B)^{30}$

$\binom{30}{27}$ $\binom{30}{28}$ $\binom{30}{29}$ $\binom{30}{30}$

$4060$

$B^{27}$ $B^{28}$ $B^{29}$ $B^{30}$ $\boxed{= 4060A^3B^{27}}$

$A^3$ $A^2$ $A^1$ $A^0$

36. Find the term containing $y^3$ in the expansion of $\left(\sqrt{2}+y\right)^{12}$

$\binom{12}{0}$ $\binom{12}{1}$ $\binom{12}{2}$ $\binom{12}{3} = 220$

$(\sqrt{2})^{12}$ $(\sqrt{2})^{11}$ $(\sqrt{2})^{10}$ $(\sqrt{2})^{9}$ $16\sqrt{2}$ $\boxed{= 3520\sqrt{2}\,y^3}$

$y^0$ $y^1$ $y^2$ $y^3$ $y^3$

39-42 Factor using the binomial theorem

42. $x^8+4x^6y+6x^4y^2+4x^2y^3+y^4$

$1$ $4$ $6$ $4$ $1$

$(x^2)^4$ $(\ )^4$ $\boxed{(x^2+y)^4}$