Geometric Sequences:

- Determine whether a sequence is Geometric
- Find the terms of an Geometric Sequence
- Express the nth term of an Geometric Sequence
- Find the partial sum of Geometric Sequence
- Find the Sum of an Infinite Geometric Series

A **Geometric Sequence** is a series of the form $a, ar, ar^2, ar^3, ar^4, \ldots$

The number ***a*** is the **first term,** and ***r*** is the **common ratio** of the sequence.
The **n^{th} term** of a geometric sequence is given by:

1-4 The nth term of a sequence is given. (a) Find the first five terms of the sequence. (b) What is the common ratio? (c) Graph the terms you found in (a)

2. $a_n = 3(-4)^{n-1}$

5-8 Find the nth term of the geometric sequence with given first term ***a*** and common ratio ***r***. What is the fourth term?

6. $a = -6, r = 3$

9-16 Determine whether the sequence is geometric. If it is geometric, find the common ratio.

10. 2, 6, 18, 36, ...

14. $e^2, e^4, e^6, e^8, \ldots$

17- 22 Find the first five terms of the sequence and determine if it is geometric. If it is geometric, find the common ratio and express the nth term of the sequence in the standard form $a_n = ar^{n-1}$.

18. $a_n = 4 + 3^n$

19. $a_n = \frac{1}{4^n}$

23- 32 Determine the common ratio, the fifth term, and the nth term of the geometric sequence.

24. $7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \ldots$

32. $5, 5^{c+1}, 5^{2c+1}, 5^{3c+1}, \ldots$

34. The first term of a geometric sequence is 3, and the third term is $\frac{4}{3}$. Find the fifth term.

38. The second and the fifth terms of a geometric sequence are 10 and 1250, respectively. Is 31,250 a term of this sequence? If so, which term is it?

Partial Sums of a Geometric Sequence:

For the geometric sequence $a_n = ar^{n-1}$, the **nth partial sum**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \quad (r \neq 1)$$

is given by

39- 42 Find the partial sum S_n of the geometric sequence that satisfies the given conditions.

42. $a_2 = 0.12, \quad a_5 = 0.00096, \quad n = 4$

43- 46. Find the sum.

46. $\sum_{j=0}^{5} 7\left(\frac{3}{2}\right)^j$

Sum of an Infinite Geometric Series:

If $|r| < 1$, then the infinite geometric series

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \quad (r \neq 1)$$ has the sum:

Find the sum of the infinite geometric series.

48. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$

55 – 60 Express the repeating decimal as a fraction.

57. 0.030303 ...

70. Geometric Savings Plan. A very patient woman wishes to become a billionaire. She decides to follow a simple scheme: She puts aside 1 cent the first day, 2 cents the second day, 4 cents the third day, and so on, doubling the number of cents each day. How much money will she have at the end of 30 days? How many days will it take this woman to realize her wish?

Notes Section 9.3

Geometric Sequences:

- Determine whether a sequence is Geometric
- Find the terms of an Geometric Sequence
- Express the nth term of an Geometric Sequence
- Find the partial sum of Geometric Sequence
- Find the Sum of an Infinite Geometric Series

A **Geometric Sequence** is a series of the form $a, ar, ar^2, ar^3, ar^4, \ldots$

The number ***a*** is the **first term,** and ***r*** is the **common ratio** of the sequence.
The **n^{th} term** of a geometric sequence is given by: $a_n = ar^{(n-1)}$

1-4 The nth term of a sequence is given. (a) Find the first five terms of the sequence. (b) What is the common ratio? (c) ~~Graph the terms you found in (a)~~

2. $a_n = 3(-4)^{n-1}$

a) 3, -12, 48, -192, 768 b) -4

5-8 Find the nth term of the geometric sequence with given first term ***a*** and common ratio ***r***. What is the fourth term?

6. $a = -6, r = 3$

$a_n = -6(3)^{n-1}$

$a_4 = -6(3)^{4-1} = -162$

9-16 Determine whether the sequence is geometric. If it is geometric, find the common ratio.

10. 2, 6, 18, 36, ...

$\frac{6}{2} = 3$ $\frac{18}{6} = 3$ $\frac{36}{18} = 2$ NO

14. $e^2, e^4, e^6, e^8, \ldots$

yes e^2

$\frac{e^4}{e^2} = e^2$ $\frac{e^6}{e^4} = e^2$ $\frac{e^8}{e^6} = e^2$

17- 22 Find the first five terms of the sequence and determine if it is geometric. If it is geometric, find the common ratio and express the nth term of the sequence in the standard form $a_n = ar^{n-1}$.

18. $a_n = 4 + 3^n$

$a_1 = 7$ $a_2 = 13$ $a_3 = 31$ $a_4 = 85$ $a_5 = 247$ NO

19. $a_n = \frac{1}{4^n}$

yes $\frac{1}{4}$

$a_1 = \frac{1}{4}$ $a_2 = \frac{1}{16}$ $a_3 = \frac{1}{64}$ $a_4 = \frac{1}{256}$ $a_5 = \frac{1}{1024}$

23- 32 Determine the common ratio, the fifth term, and the nth term of the geometric sequence.

24. $7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \ldots$ $\frac{112}{81}$

$r = \frac{\frac{14}{3}}{7} = \frac{14}{3} \cdot \frac{1}{7} = \frac{2}{3}$

$a_n = 7\left(\frac{2}{3}\right)^{n-1}$

32. $5, 5^{c+1}, 5^{2c+1}, 5^{3c+1}, \ldots$ 5^{4c+1}

$r = \frac{5^{c+1}}{5^1} = 5$

$a_n = 5(5^c)^{n-1}$

34. The first term of a geometric sequence is 3, and the third term is $\frac{4}{3}$. Find the fifth term.

$\frac{\frac{4}{3}}{3} = \frac{4}{9} = r^2$ $\quad \frac{3}{1^{st}}, \_\_, \frac{\frac{4}{3}}{3^{rd}}$ $\quad r = \frac{2}{3}$ $\quad a_n = 3\left(\frac{2}{3}\right)^{n-1}$

$a_5 = 3\left(\frac{2}{3}\right)^4 = \frac{16}{27}$

38. The second and the fifth terms of a geometric sequence are 10 and 1250, respectively. Is 31,250 a term of this sequence? If so, which term is it?

$\frac{10}{2^{nd}}, \_\_, \_\_, \frac{1250}{5^{th}}$ $\quad r, r, r$

$r^3 = \frac{1250}{10} = 125$ $\quad r = 5$ $\quad a_n = 2(5)^{n-1}$

$31{,}250 = 2(5)^{n-1}$

$15625 = 5^{n-1}$

$15625 = 5^6$

$n - 1 = 6$

$n = 7$

Partial Sums of a Geometric Sequence:

For the geometric sequence $a_n = ar^{n-1}$, the **nth partial sum**

$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1}$ $(r \neq 1)$

is given by $S_n = a\left(\frac{1-r^n}{1-r}\right)$

39- 42 Find the partial sum S_n of the geometric sequence that satisfies the given conditions.

42. $a_2 = 0.12, \quad a_5 = 0.00096, \quad n = 4$

$r^3 = \frac{.00096}{.12} = .008$ $\quad r = .2$ $\quad a = .6$

$S_n = .6\left(\frac{1-.2^4}{1-.2}\right)$

$.7488$

43- 46. Find the sum.

46. $\sum_{j=0}^{5} 7\left(\frac{3}{2}\right)^j$ $\quad r = \frac{3}{2}$ $\quad a = 7$ $\quad n = 6$

$S_n = 7\left(\frac{1-\left(\frac{3}{2}\right)^6}{1-\frac{3}{2}}\right) = \frac{4655}{32}$

Sum of an Infinite Geometric Series:

If $|r| < 1$, then the infinite geometric series

$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1}$ $(r \neq 1)$ has the sum:

$S_n = \frac{a}{1-r}$

Find the sum of the infinite geometric series.

48. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$

$r = -\frac{1}{2}$ $\quad S_n = \frac{1}{1--\frac{1}{2}} = \frac{2}{3}$

55 – 60 Express the repeating decimal as a fraction.

57. 0.030303 ...

$.03 + .0003 + .000003$ $\quad r = \frac{1}{100}$

$\frac{3}{100} + \frac{3}{10{,}000} + \frac{3}{1{,}000{,}000}$ $\quad a = \frac{3}{100}$

$S_n = \frac{\frac{3}{100}}{1-\frac{1}{100}}$

$= \frac{3}{99} = \frac{1}{33}$

$= \frac{3}{100} \cdot \frac{100}{99}$

70. Geometric Savings Plan. A very patient woman wishes to become a billionaire. She decides to follow a simple scheme: She puts aside 1 cent the first day, 2 cents the second day, 4 cents the third day, and so on, doubling the number of cents each day. How much money will she have at the end of 30 days? How many days will it take this woman to realize her wish?

$a = 1$ $\quad r = 2$ $\quad S_n = 1\left(\frac{1-2^{30}}{1-2}\right) = 1{,}073{,}741{,}823 \div 10$

$10^9 = .01\left(\frac{1-2^n}{1-2}\right) = 10^{11} - 1 = 2^n$ $\quad n = 33$ days $\quad 10{,}737{,}418.23$