Notes 7.3
Inverses of Matrices

Solve systems of equations using:

- Find the Inverse of a Matrix
- Use the Inverse of a Matrix to solve an equation

The identity Matrix:

The inverse of a 2 X 2 Matrix:

1-4 Calculate the products AB and BA to verify that B is the inverse of A:

2. $A = \begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$, $B = \begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ 2 & -1 \end{bmatrix}$

7-22 Find the inverse of the matrix if it exists:

8. $\begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix}$

23-30 Solve the system of equations by converting to a matrix equation and using the inverse coefficient matrix. Use the inverses from Exercises 7-10, 15, 16, 19, 21.

24. $\begin{cases} 3x + 4y = 10 \\ 7x + 9y = 20 \end{cases}$

7-22 Find the inverse of the matrix if it exists:

16. $\begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix}$

23-30 Solve the system of equations by converting to a matrix equation and using the inverse coefficient matrix. Use the inverses from Exercises 7-10, 15, 16, 19, 21.

28. $\begin{cases} 5x + 7y + 4z = 1 \\ 3x - y + 3z = 1 \\ 6x + 7y + 5z = 1 \end{cases}$

Calculator Portion:

31-36 Use a calculator that can perform matrix operations to solve the system.

32. $\begin{cases} 3x + 4y - z = 2 \\ 2x - 3y + z = -5 \\ 5x - 2y + 2z = -3 \end{cases}$

Notes 7.3
Inverses of Matrices

Solve systems of equations using:

- Find the Inverse of a Matrix
- Use the Inverse of a Matrix to solve an equation

The identity Matrix: $A \cdot I = A$ or $I \cdot A = A$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ← diagonal is ones with zero everywhere else

The inverse of a 2 X 2 Matrix:

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$; $ad-bc \neq 0$

1-4 Calculate the products AB and BA to verify that B is the inverse of A:

2. $A = \begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$, $B = \begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ 2 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 7/2 & -3/2 \\ 2 & -1 \end{bmatrix}$ $A = \begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7-22 Find the inverse of the matrix if it exists:

8. $\begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix}$

$\frac{1}{27-28}\begin{bmatrix} 9 & -4 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -9 & 4 \\ 7 & -3 \end{bmatrix}$

23-30 Solve the system of equations by converting to a matrix equation and using the inverse coefficient matrix. Use the inverses from Exercises 7-10, 15, 16, 19, 21.

24. $\begin{cases} 3x + 4y = 10 \\ 7x + 9y = 20 \end{cases}$

$\begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

A X B

$AX = B$

$X = A^{-1}B$

X A^{-1}

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 & 4 \\ 7 & -3 \end{bmatrix}\begin{bmatrix} 10 \\ 20 \end{bmatrix}$

$= \begin{bmatrix} -10 \\ 10 \end{bmatrix}$

Check: $3(-10) + 4(10) = 10$
$7(-10) + 9(10) = 20$

7-22 Find the inverse of the matrix if it exists:

16. $\begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 0 & 9 & -1 & 0 & -2 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 5 & 7 & 4 & 1 & 0 & 0 \\ 0 & 26 & -3 & 3 & -5 & 0 \\ 0 & 9 & -1 & 0 & -2 & 1 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 5 & 7 & 4 & 1 & 0 & 0 \\ 0 & 26 & -3 & 3 & -5 & 0 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right] = \left[\begin{array}{ccc|ccc} 5 & 7 & 0 & 109 & 28 & -104 \\ 0 & 26 & 0 & -78 & -26 & 78 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right] = \left[\begin{array}{ccc|ccc} 5 & 7 & 0 & 109 & 28 & -104 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 5 & 0 & 0 & 130 & 35 & -125 \\ 0 & 1 & 0 & -3 & 1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right]$

23-30 Solve the system of equations by converting to a matrix equation and using the inverse coefficient matrix. Use the inverses from Exercises 7-10, 15, 16, 19, 21.

28. $\begin{cases} 5x + 7y + 4z = 1 \\ 3x - y + 3z = 1 \\ 6x + 7y + 5z = 1 \end{cases}$ $\begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1} B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -7 & 26 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 8 \end{bmatrix}$

Check $5(8) + 7(-1) + 4(8) = 1$ ✓

$3(8) - (-1) + 3(8) = 1$ ✓

$6(8) + 7(-1) + 5(8) = 1$ ✓

Calculator Portion:

31-36 Use a calculator that can perform matrix operations to solve the system.

32. $\begin{cases} 3x + 4y - z = 2 \\ 2x - 3y + z = -5 \\ 5x - 2y + 2z = -3 \end{cases}$ $\begin{bmatrix} 3 & 4 & -1 \\ 2 & -3 & 1 \\ 5 & -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$

$A \cdot X = B$

$X = A^{-1} \cdot B$

$\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$3(-1) + 4(2) - 3 = 2$ ✓

$2(-1) - 3(2) + 3 = -5$ ✓

$5(-1) - 2(2) + 2(3) = -3$ ✓