Notes 6.5 Partial Fractions

Decompose Fractions into the sum of partial fractions

- **When denominators are linear (non-repeating)**
- **When denominators are linear (repeating)**
- **When the denominators are quadratic (non-repeating)**
- **When the denominators are quadratic (repeating)**

What are Partial Fractions? $\frac{2}{x-2}+\frac{3}{x+1}=$

But how do we go in the opposite direction?

$$\frac{5x-4}{x^2-x-2}$$

Step 1: ______________________________________

Step 2: What kinds of factors are in the denominator?

Separate partial fractions according to the factors:

Factor in denominator	***Term in partial fraction decomposition***
$ax+b$	
$(ax+b)^n$	
ax^2+bx+c	
$(ax^2+bx+c)^n$	

1-10 Write the form of the partial fraction decomposition of the function:

Do not determine the numerical value of the coefficients: $\boldsymbol{ax+b}$

2. $\frac{x}{x^2+3x-4}$

Step 3: Multiply through by the denominator so we no longer have fractions:

11-42 Find the partial fraction decomposition of the rational function:

18. $\frac{2x+1}{x^2+x-2}$

22. $\frac{7x-3}{x^3+2x^2-3x}$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients: $(ax + b)^n$

4. $\frac{1}{x^4 - x^3}$

11-42 *Find the partial fraction decomposition of the rational function:*

27. $\frac{2x}{4x^2+12x+9}$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients: $ax^2 + bx + c$

6. $\frac{1}{x^4 - 1}$

11-42 *Find the partial fraction decomposition of the rational function:*

36. $\frac{3x^2-2x+8}{x^3-x^2+2x-2}$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients:
$(ax^2 + bx + c)^n$

8. $\frac{x^4+x^2+1}{x^2(x^2+4)^2}$

11-42 *Find the partial fraction decomposition of the rational function:*

40. $\frac{2x^2-x+8}{(x^2+4)^2}$

Notes 6.5 Partial Fractions

Decompose Fractions into the sum of partial fractions

- **When denominators are linear (non-repeating)**
- **When denominators are linear (repeating)**
- **When the denominators are quadratic (non-repeating)**
- **When the denominators are quadratic (repeating)**

What are Partial Fractions?

$$\frac{2}{x-2}+\frac{3}{x+1}=\frac{2(x+1)+3(x-2)}{(x-2)(x+1)}=\frac{5x-4}{x^2-x-2}$$

But how do we go in the opposite direction?

$$\frac{5x-4}{x^2-x-2}=\frac{A}{(x-2)}+\frac{B}{(x+1)}$$

Step 1: Factor the Denominator

Step 2: What kinds of factors are in the denominator?
Separate partial fractions according to the factors:

	Factor in denominator	*Term in partial fraction decomposition*
Linear factors	$ax+b$	$\boxed{\frac{A}{ax+B}}$
	$(ax+b)^n$	$\frac{A}{(ax+b)}+\frac{B}{(ax+b)^2}+\cdots+\frac{C}{(ax+b)^n}$
Quadratic	ax^2+bx+c	$\boxed{\frac{Ax+B}{ax^2+bx+c}}$
	$(ax^2+bx+c)^n$	$\frac{Ax+B}{(ax^2+bx+c)}+\frac{Cx+D}{(ax^2+bx+c)^2}+\cdots+\frac{Ex+F}{(ax^2+bx+c)^n}$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients: $\boldsymbol{ax+b}$

2. $\frac{x}{x^2+3x-4}=\frac{x}{(x+4)(x-1)}=\frac{A}{(x+4)}+\frac{B}{(x-1)}$

Step 3: Multiply through by the denominator so we no longer have fractions:

11-42 *Find the partial fraction decomposition of the rational function:*

18. $\frac{2x+1}{x^2+x-2}=\frac{2x+1}{(x+2)(x-1)}$

$=\frac{A}{(x+2)}+\frac{B}{(x-1)}=\boxed{\frac{1}{x+2}+\frac{1}{x-1}}$

$A(x-1)+B(x+2)=Ax-A+Bx+2B$

$A+B=2$, $-A+2B=1$

$A+B=2$

$3B=3$

$B=1$

$A=1$

22. $\frac{7x-3}{x^3+2x^2-3x}=\frac{7x-3}{x(x^2+2x-3)}=\frac{7x-3}{x(x+3)(x-1)}$

$\frac{A}{x}+\frac{B}{x+3}+\frac{C}{x-1}=\boxed{\frac{1}{x}-\frac{2}{x+3}+\frac{1}{x-1}}$

$A(x^2+2x-3)+B(x^2-x)+C(x^2+3x)=7x-3$

$A+B+C=0$; $2A-B+3C=7$; $-3A=-3$

$A=1$

$B+C=-1$

$-B+3C=5$

$4C=4$

$C=1$

$B=-2$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients: $(ax+b)^n$

4. $\frac{1}{x^4-x^3} = \frac{1}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1}$

11-42 *Find the partial fraction decomposition of the rational function:*

27. $\frac{2x}{4x^2+12x+9} = \frac{2x}{(2x+3)^2} = \frac{A}{(2x+3)} + \frac{B}{(2x+3)^2} = \boxed{\frac{1}{2x+3} - \frac{3}{(2x+3)^2}}$

$A(2x+3)+B = 2x$

$2Ax + 3A + B = 2x$

$2A = 2 \quad A = 1 \qquad 3A + B = 0 \quad B = -3$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients: ax^2+bx+c

6. $\frac{1}{x^4-1} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x-1)(x+1)(x^2+1)}$

$= \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{Cx+D}{(x^2+1)}$

11-42 *Find the partial fraction decomposition of the rational function:*

36. $\frac{3x^2-2x+8}{x^3-x^2+2x-2} = \frac{3x^2-2x+8}{x^2(x-1)+2(x-1)} = \frac{3x^2-2x+8}{(x^2+2)(x-1)}$

$\frac{Ax+B}{x^2+2} + \frac{C}{(x-1)} = \boxed{\frac{-2}{x^2+2} + \frac{3}{x-1}}$

$(Ax+B)(x-1) + C(x^2+2) = 3x^2-2x+8$

$Ax^2 - Ax + Bx - B + Cx^2 + 2C = 3x^2 - 2x + 8$

$A + C = 3$
$-A + B = -2$
$B + C = 1$
$-B + 2C = 8$
$B + C = 1$
$3C = 9$
$C = 3;\ B = -2;\ A = 0$

1-10 *Write the form of the partial fraction decomposition of the function:*
Do not determine the numerical value of the coefficients:
$(ax^2+bx+c)^n$

8. $\frac{x^4+x^2+1}{x^2(x^2+4)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x^2+4)} + \frac{Ex+F}{(x^2+4)^2}$

11-42 *Find the partial fraction decomposition of the rational function:*

40. $\frac{2x^2-x+8}{(x^2+4)^2} = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2} = \boxed{\frac{2}{(x^2+4)} - \frac{x}{(x^2+4)^2}}$

$(Ax+B)(x^2+4) + Cx + D = 2x^2 - x + 8$

$Ax^3 + Bx^2 + 4A + 4B + Cx + D = 2x^2 - x + 8$

$A = 0 \quad B = 2 \quad 4A + C = -1 \quad 4B + D = 8$

$C = -1 \quad D = 0$