

## 7.4- Graphing Rational Functions

I can graph the parent function  $y = \frac{1}{x}$

I can use transformations to graph any reciprocal function.

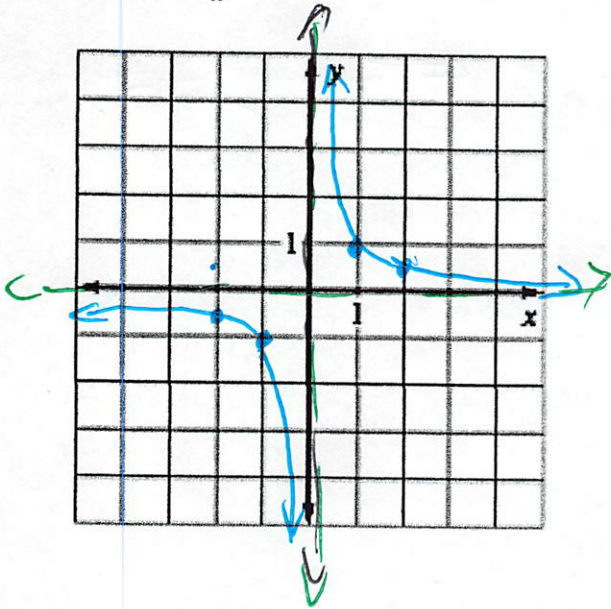
I can identify the vertical and horizontal asymptotes of a reciprocal function.

I can identify the domain and range of a reciprocal function

### Vocabulary:

- Reciprocal function → a function that has an equation of the form  $f(x) = \frac{1}{a(x)}$ , where  $a(x)$  is a linear function and  $a(x) \neq$  zero
- Hyperbola → the name given to the graph of a reciprocal function
- Asymptote → a line that the graph of a function approaches

Graph:  $y = \frac{1}{x}$



Asymptotes:  $x = 0$   
 $y = 0$

D:  $x \neq 0$   
 $y \neq 0$

make a table

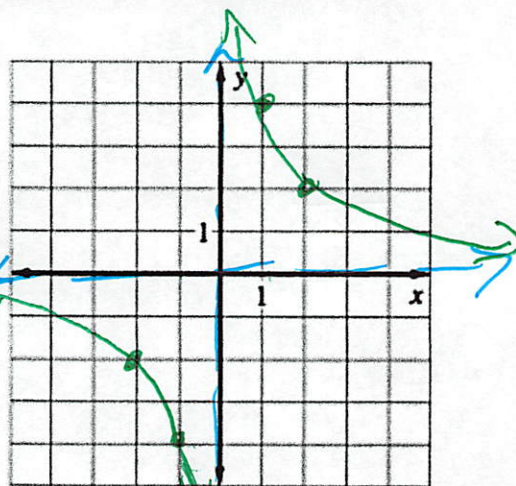
x	y
-2	-1/2
-1	-1
1	1
2	1/2

### PARENT FUNCTION FOR SIMPLE RATIONAL FUNCTIONS

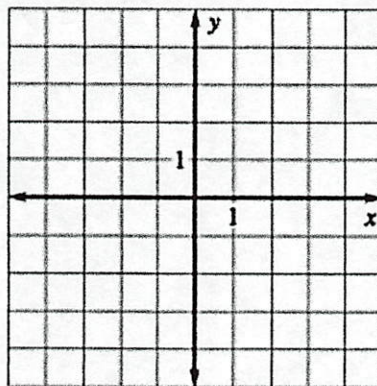
- The graph of the parent function  $f(x) = \frac{1}{x}$  is a hyperbola, which consists of two symmetrical parts called branches.
- The domain and range are all nonzero real numbers.
- The asymptotes are  $x = 0$ , and  $y = 0$ .
- Any function of the form  $g(x) = \frac{a}{x}$  ( $a \neq 0$ ) has the same asymptotes, domain, and range as the function  $f(x) = \frac{1}{x}$ .

**Example 1**Graph a rational function of the form  $y = \frac{a}{x}$ Graph the function  $y = \frac{4}{x}$ .1. Draw the asymptotes  $x = \underline{0}$  and  $y = \underline{0}$ .2. Plot points to the left and to the right of the vertical asymptote, such as  $(-2, \underline{-2})$ ,  $(-1, \underline{-4})$ ,  $(1, \underline{4})$ , and  $(2, \underline{2})$ .

3. Draw the branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

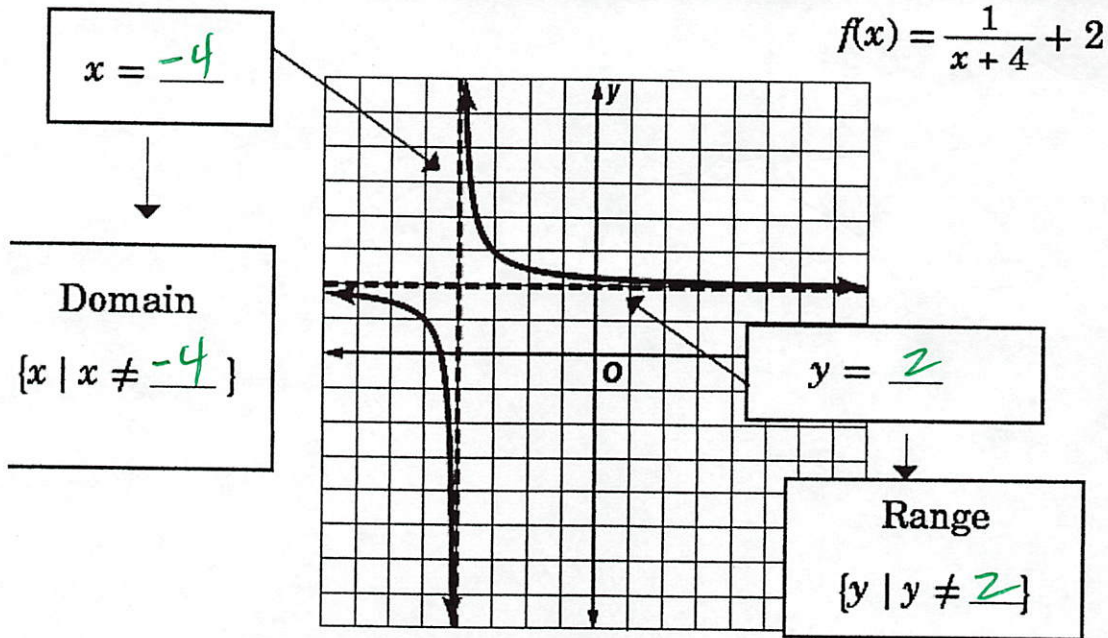
 $D: x \neq 0$   $R: y \neq 0$ ✔ **Checkpoint** Graph the function.

1.  $y = \frac{3}{x}$

**GRAPHING TRANSLATIONS OF SIMPLE RATIONAL FUNCTIONS**To graph a rational function of the form  $y = \frac{a}{x-h} + k$ , follow these steps:**Step 1** Draw the asymptotes  $x = \underline{h}$  and  $y = \underline{k}$ .**Step 2** Plot points to the left and to the right of the vertical asymptote.**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

Vertical and Horizontal Asymptotes:

Sketch the vertical and horizontal asymptotes in the graph below. Write the equations of the asymptotes and then write the domain and range for the function.



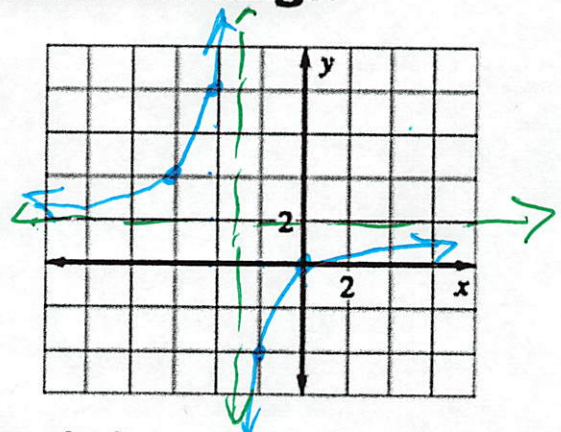
**Example 2**

Graph a rational function  $y = \frac{a}{x-h} + k$

Graph  $y = \frac{-6}{x+3} + 2$ . State the domain and range.

1. Draw the asymptotes  $x = -3$  and  $y = 2$ .

2. Plot points to the left and to the right of the vertical asymptote, such as  $(-6, 4)$ ,  $(-4, 8)$ ,  $(-2, -4)$ , and  $(0, 0)$ .



3. Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

The domain is  $x \neq -3$ , and the range is  $y \neq 2$ .

## Transformations of Reciprocal Functions:

Compare and contrast the graphs of  $f(x) = \frac{1}{x}$  to  $g(x) = \frac{3}{x-2} - 5$  for each of the listed characteristics.

	Domain	Range	Vertical Asymptotes	Horizontal Asymptotes
$f(x)$	$x \neq 0$	$y \neq 0$	$x = 0$	$y = 0$
$g(x)$	$x \neq 2$	$y \neq -5$	$x = 2$	$y = -5$

✔ **Checkpoint** Graph the function. State the domain and range.

2.  $y = \frac{3}{x-2} + 1$

