Notes 7.2
The Algebra of Matrices

- Sum and Difference of Matrices
- Scalar Product of Matrices
- Matrix Multiplication.

1-2 Determine whether the matrices A and B are equal.

2. $A = \begin{bmatrix} \frac{1}{4} & \ln 1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.25 & 0 \\ \sqrt{4} & \frac{6}{2} \end{bmatrix}$

ADDING AND SUBTRACTING MATRICES

To add or subtract two matrices, simply add or subtract _____ elements. You can add or subtract matrices only if they have the same _____.

Adding Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

MULTIPLYING MATRICES

Words To find the element in the *i*th row and *j*th column of the product matrix *AB*, multiply each element in the _____ by the corresponding element in the _____, then add the products.

Algebra $\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{AB}{\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}}$

3-10 Perform the matrix operation, or if it is possible, explain why.

4. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & -2 \end{bmatrix}$

6. $2\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

3-10 Perform the matrix operation, or if it is possible, explain why.

8. $\begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

11-16 Solve the matrix equation for the unknown matrix X, or explain why no solution exists.

$A = \begin{bmatrix} 4 & 6 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 10 & 20 \\ 30 & 20 \\ 10 & 0 \end{bmatrix}$

12. $3X - B = C$

14. $5(X - C) = D$

17–38 ■ The matrices A, B, C, D, E, F, and G are defined as follows.

$$A = \begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 3 & \frac{1}{2} & 5 \\ 1 & -1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & -\frac{5}{2} & 0 \\ 0 & 2 & -3 \end{bmatrix}$$

$$D = \begin{bmatrix} 7 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} 5 & -3 & 10 \\ 6 & 1 & 0 \\ -5 & 2 & 2 \end{bmatrix}$$

Carry out the indicated algebraic operation, or explain why it cannot be performed.

22. $C - 5A$ 24. DA 26. BC

39-42 Solve for x and y:

40. $3\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ -9 & 6 \end{bmatrix}$

43-46 Write the system of equations as a matrix equation.(As in example 6)

44.
$$\begin{cases} 6x - y + z = 12 \\ 2x + z = 7 \\ y - 2z = 4 \end{cases}$$

47. Let

$$A = \begin{bmatrix} 1 & 0 & 6 & -1 \\ 2 & \frac{1}{2} & 4 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 7 & -9 & 2 \end{bmatrix}$$

Determine which of the following products are defined, and calculate the ones that are:

ABC ACB BAC

BCA CAB CBA

Notes 7.2
The Algebra of Matrices

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1-2 Determine whether the matrices A and B are equal.

2. $A = \begin{bmatrix} \frac{1}{4} & \ln 1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0.25 & 0 \\ \sqrt{4} & \frac{6}{2} \end{bmatrix}$

$\frac{1}{4} = .25$ $\quad \ln 1 = 0$

$2 = \sqrt{4}$ $\quad 3 = \frac{6}{2}$ $\quad$ yes.

ADDING AND SUBTRACTING MATRICES

To add or subtract two matrices, simply add or subtract corresponding elements. You can add or subtract matrices only if they have the same dimensions

Adding Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Subtracting Matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

MULTIPLYING MATRICES

Words To find the element in the *i*th row and *j*th column of the product matrix *AB*, multiply each element in the i^{th} Row A by the corresponding element in the j^{th} column B, then add the products.

Algebra $\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{AB}{\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}}$

3-10 Perform the matrix operation, or if it is possible, explain why.

4. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 3 \end{bmatrix}$

6. $2\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$ not possible different dimensions

3-10 Perform the matrix operation, or if it is possible, explain why.

8. $\begin{bmatrix} 2 & 1 & 2 \\ 6 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 6 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 2+3+-4, & -4+6+0 \\ 6+9+-8, & -12+18+0 \end{bmatrix}$

$(2\times3)(3\times2) = 2\times2$ $\quad = \begin{bmatrix} 2 & 2 \\ 7 & 6 \end{bmatrix}$

10. $\begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 10+-3 \\ 0+1 \\ 5+2 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 7 \end{bmatrix}$

$(3\times2)(2\times1) = (3\times1)$

11-16 Solve the matrix equation for the unknown matrix X, or explain why no solution exists.

$A = \begin{bmatrix} 4 & 6 \\ 1 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$ $\quad C = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad D = \begin{bmatrix} 10 & 20 \\ 30 & 20 \\ 10 & 0 \end{bmatrix}$

12. $3X - B = C$

$3X - B = C$

$3X = B + C$

$X = \frac{1}{3}[B+C]$

Can't be done

14. $5(X - C) = D$

$X - C = \frac{D}{5}$

$X = \frac{D}{5} + C$

$X = \frac{1}{5}\begin{bmatrix} 10 & 20 \\ 30 & 20 \\ 10 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 7 & 4 \\ 2 & 2 \end{bmatrix}$

17–38 ■ The matrices A, B, C, D, E, F, and G are defined as follows.

$$A=\begin{bmatrix}2 & -5\\ 0 & 7\end{bmatrix} \quad B=\begin{bmatrix}3 & \frac{1}{2} & 5\\ 1 & -1 & 3\end{bmatrix} \quad C=\begin{bmatrix}2 & -\frac{5}{2} & 0\\ 0 & 2 & -3\end{bmatrix}$$

$$D=\begin{bmatrix}7 & 3\end{bmatrix} \quad E=\begin{bmatrix}1\\ 2\\ 0\end{bmatrix} \quad F=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad G=\begin{bmatrix}5 & -3 & 10\\ 6 & 1 & 0\\ -5 & 2 & 2\end{bmatrix}$$

Carry out the indicated algebraic operation, or explain why it cannot be performed.

22. $C-5A$

Can't do diff. dimensions

24. DA

$\begin{bmatrix}7 & 3\end{bmatrix}\cdot\begin{bmatrix}2 & -5\\ 0 & 7\end{bmatrix}=\begin{bmatrix}14+0, & -35+21\end{bmatrix}$

$=[14, -14]$

26. BC

2x3 2x3

Can't multiply Columns & Rows don't match

39-42 Solve for x and y:

40. $3\begin{bmatrix}x & y\\ y & x\end{bmatrix}=\begin{bmatrix}6 & -9\\ -9 & 6\end{bmatrix}$

$3x=6$ $\quad x=2$ $\quad y=-9$

43-46 Write the system of equations as a matrix equation.(As in example 6)

44. $\begin{cases}6x-y+z=12\\ 2x+z=7\\ y-2z=4\end{cases}$

$$\begin{bmatrix}6 & -1 & 1\\ 2 & 0 & 1\\ 0 & 1 & -2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}12\\ 7\\ 4\end{bmatrix}$$

47. Let

$$A=\begin{bmatrix}1 & 0 & 6 & -1\\ 2 & \frac{1}{2} & 4 & 0\end{bmatrix} \quad C=\begin{bmatrix}1\\ 0\\ -1\\ -2\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 7 & -9 & 2\end{bmatrix}$$

Determine which of the following products are defined, and calculate the ones that are:

ABC	ACB	BAC
BCA	CAB	CBA

ABC N.P.

ACB

2x4 4x1

2x1 1x4 = 2x4

AC · B

ACB

$$A=\begin{bmatrix}1 & 0 & 6 & -1\\ 2 & \frac{1}{2} & 4 & 0\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\\ -1\\ -2\end{bmatrix}=\begin{bmatrix}1+0+-6+2\\ 2+0+-4+0\end{bmatrix}=\begin{bmatrix}-3\\ -2\end{bmatrix}\cdot\begin{bmatrix}1 & 7 & -9 & 2\end{bmatrix}-3\cdot$$

$$ACB=\begin{bmatrix}-3 & -21 & 27 & -6\\ -2 & -14 & 18 & -4\end{bmatrix}$$