Notes 6.6 –_Using Exponential and Logarithmic Functions – Applications

I can solve half-life problems
I can solve exponential growth/decay story problems

Exponential Growth and Decay

Exponential Growth	$f(x) = ae^{kt}$ where a is the initial value of y, t is time in years, and k is a constant representing the **rate of continuous growth.**
Exponential Decay	$f(x) = ae^{-kt}$ where a is the initial value of y, t is time in years, and k is a constant representing **the rate of continuous decay.**

1- POPULATION In 2000, the world population was estimated to be 6.124 billion people. In 2005, it was 6.515 billion.

$t = 2005 - 2000 = 5$, $a = 6.124$

a. Determine the value of k, the world's relative rate of growth. the value of k, the world's relative rate of growth.

ae^{kt}

$6.515 = 6.124e^{k(5)}$ $\qquad \frac{6.515}{6.124} = e^{k(5)}$

$\ln\left(\frac{6.515}{6.124}\right) = 5k$

$\boxed{.01238 = k}$

b. When will the world's population reach 7.5 billion people?

$7.5 = 6.124e^{(.01238)t}$ $\qquad \frac{7.5}{6.124} = e^{.01238t}$

$t = 16.4$ years

2. CARBON DATING Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of carbon 14, t is the number of years ago the animal lived, and y is the remaining amount after t years.

a. If the initial amount on carbon 14 is 2800, how long will it take to reach 150?

$150 = 2800e^{-.00012t}$ $\qquad e^{-.00012t} = \frac{150}{280}$

$\boxed{t = 24{,}390 \text{ years}}$

b. How much Carbon-14 remains after 2000 years?

$y = 2800e^{-.00012(2000)}$ $\qquad \boxed{2203}$

3. BIOLOGY A certain bacteria is growing exponentially according to the model $y = ae^{kt}$, a is the initial amount of bacteria, t is the time in minutes, and y is the remaining amount after t minutes.

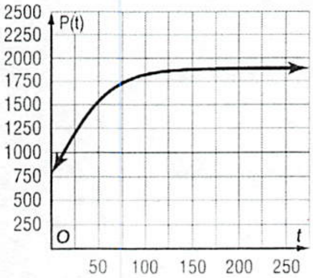
a. If there are 80 cells initially and 675 cells after 30 minutes, find the value of k for the bacteria.

b. When will the bacteria reach a population of 6000 cells?

4- Logistic Growth A logistic function models the S-curve of growth of some set λ. The initial stage of growth is approximately exponential; then, as saturation begins, the growth slows, and at some point, growth stops.

The population of a certain species of fish in a lake after *t* years is given by $P(t) = \frac{1880}{1 + 1.42e^{-0.037t}}$.

a. What is maximum population?



1880

b. When will the population reach 1875?

$1875 = \frac{1880}{1+1.42e^{-.037t}}$

$1875(1+1.42e^{-.037t}) = 1880$

$1+1.42e^{-.037t} = \frac{1880}{1875}$

$\boxed{170 \text{ years}}$

5. LOGISTIC GROWTH The population of a bacteria can be modeled by $P(t) = \frac{22{,}000}{1 + 1.2e^{-0.0971t}}$ where *t* is time in hours and *k* is a constant.

a. What is the maximum population?

b. When does the population reach 21,000?

6. CARBON DATING Archeologists uncover an ancient wooden tool. They analyze the tool and find that it has 22% as much carbon 14 compared to the likely amount that it contained when it was made. Given that the half-life of carbon 14 is about 5730 years, about how old is the artifact? Round your answer to the nearest 100 years.

$\frac{1}{2} = e^{k(5730)}$

$k \approx -.000121$

$.22 = e^{-.000121t}$

$\ln(.22) = -.000121t$

$t \approx 12{,}500$ years

7. HALF-LIFE. I-123 is used in thyroid scans. If the half-life is 13.2 hours, find the value of k for I-123.

$\frac{1}{2} = e^{13.2k}$

$\ln\frac{1}{2} = 13.2k$

$k \approx -0.0525$

8. RADIOACTIVE DECAY A radioactive substance has a half-life of 40 years. Find the constant *k* in the decay formula for the substance.