Notes 6.5 – Base e and Natural Logarithms

I understand what a natural logarithm is
I can solve natural log problems using base e
I can simplify expressions with e and the natural log
I can solve base e equations
I can solve story problems with continuously compounded interest.

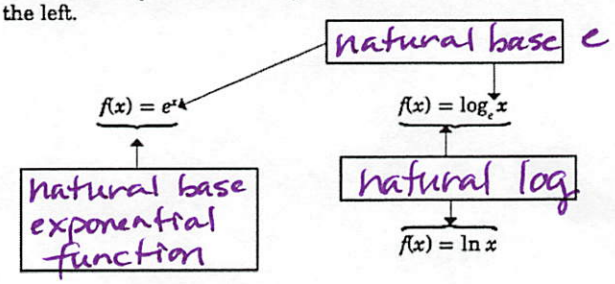
THE NATURAL BASE *e*

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1+\frac{1}{n}\right)^n$ approaches

$e \approx$ _____.

New Vocabulary Label the diagram with the terms listed at the left.



Simplify natural base expressions:

a. $e^6 \cdot e^3 = e^9$ b. $\frac{18e^6}{2e^4} = 9e^2$ c. $(4e^{3x})^2 = 16e^{6x}$

Evaluate natural base expressions:

Use a calculator to evaluate the expression:

a. $e^{-2} = 0.135$ b. $e^{0.3} = 1.350$

* 1. Simplify $e^{-3} \cdot e^6$ 2. Simplify $\frac{(4e^3)^2}{8e^5}$

Use a calculator to evaluate the expression:

3. e^{-3} 4. $e^{0.36}$.

Write an equivalent base e exponential equation for a natural logarithmic equation by using:

$\ln x = \log_e x$

$\ln 4 = x \rightarrow \log_e 4 = x \rightarrow e^x = 4$

a. $\ln x = 3$ b. $e^x = 38$

$\log_e x = 3 \rightarrow e^3 = x$ $x = \ln 38$

Write each expression in another form, then use a calculator to evaluate. Check your answer by substituting into the original expression.

$e^x = 15$	$x = \ln 15$	$x \approx 2.708$	$e^{(ans)} = 15$
$e^x = -22$	$x = \ln(-22)$	No sol.	
$\ln x = 2$	$x = e^2$	$x \approx 7.389$	$\ln(ans) = 2$
$\ln x = -5$	$x = e^{-5}$	$x \approx 0.007$	$\ln(ans) = -5$

Use answer key on Calculator

Simplify Expressions with *e* and the Natural Log:

Write each expression as a single logarithm.

a. $3\ln 10 - \ln 8 =$ $\ln 10^3 - \ln 8 = \ln \frac{10^3}{8} = \ln 125 = \ln 5^3 = 3\ln 5$

You try ☺

1. $3\ln 3 - \ln 9$

2. $4\ln 16 - \ln 256$

3. $2\ln x + 2\ln 4$

4. $3\ln 4 + 3\ln 3$

Solve Base *e* equations:

$4e^{-2x} - 5 = 3$ $4e^{-2x} = 8$ $e^{-2x} = 2$ $-2x = \ln 2$ $x \approx -0.347$

$6e^{0.25x} + 8 = 20$ $6e^{0.25x} = 12$ $e^{0.25x} = 2$ $0.25x = \ln 2$ $x = \frac{\ln 2}{0.25}$ $x \approx 2.773$

You try ☺

1. $3^{7x-3} = 9^{2x}$

2. $5^x = 72$

3. $8^{3x+2} - 6 = 5$

4. $3e^{0.5x} + 2 = 5$

Solve Base *e* inequalities:

$2e^x \leq 17.4$ $e^x < 8.7$ $x < \ln 8.7$ $x < 2.163$

$-4\ln x^3 > -72$ $\ln x^3 < 18$ $3\ln x < 18$ $\ln x < 6$ $x < e^6$ $x < 403.43$

You try ☺

$-3e^x \leq -15.6$

$7\ln x^2 < 83$

Continuously Compounded Interest:

$A = Pe^{rt}$

Describe each variable and identify its value from the real-world problem.

Ming-Na puts $600 in a savings account in which interest is compounded continuously. How much money will she have after 5 years if the annual interest rate is 4%?

How much money? $A = Pe^{rt}$ .04 (change % to a decimal) 600 5

$600e^{(.04)(5)} = \$732.84$