Notes 6.1 Solving Systems of Equations (linear and non-linear)

Solve systems of equations using:

- The substitution method.
- The elimination method.
- Graphing without a calculator.
- Graphing with a calculator.

THE SUBSTITUTION METHOD

Step 1 Solve one of the equations for one of its variables.

Step 2 Substitute the expression from ________ into the other equation and solve for the other variable.

Step 3 Substitute the value from ________ into the revised equation from Step 1 and solve.

1-8 Use the substitution method to find all solutions of the system of equations.

8. $\begin{cases} x^2 - y = 1 \\ 2x^2 + 3y = 17 \end{cases}$

THE ELIMINATION METHOD

Step 1 **Multiply** one or both of the equations by a __________ to obtain coefficients that differ only in ______ for one of its variables.

Step 2 **Add** the revised equations from ________. Combining like terms will __________ one of the variables. Solve for the remaining variable.

Step 3 **Substitute** the value obtained in ________ into either of the original equations and solve for the other variable.

9-16 Use the elimination method to find all solutions of the system of equations.

12. $\begin{cases} 3x^2 + 4y = 17 \\ 2x^2 + 5y = 2 \end{cases}$

Solve the linear system using the substitution method:

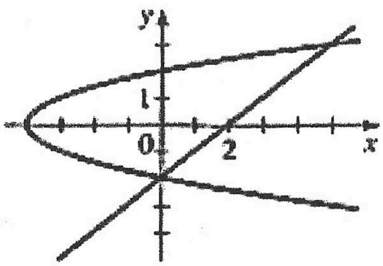
$$\begin{aligned} 2x + y &= -2 \\ 5x + 3y &= -8 \end{aligned}$$

Solve the linear system using the elimination method:

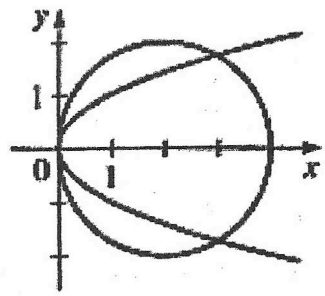
$$\begin{aligned} 3x + 8y &= -5 \\ -2x + 2y &= 18 \end{aligned}$$

17-22 Two equations and their graphs are given. Find the intersection point(s) of the graphs by solving the system.

20. $\begin{cases} x - y^2 = -4 \\ x - y = 2 \end{cases}$



22. $\begin{cases} x^2 + y^2 = 4x \\ x = y^2 \end{cases}$



23-36 Find all solutions of the system of equations.

25. $\begin{cases} x - 2y = 2 \\ y^2 - x^2 = 2x + 4 \end{cases}$

36. $\begin{cases} \dfrac{4}{x^2} + \dfrac{6}{y^4} = \dfrac{7}{2} \\ \dfrac{1}{x^2} - \dfrac{2}{y^4} = 0 \end{cases}$

49. Dimensions of a Rectangle The perimeter of a rectangle is 70 and its diagonal is 25. Find its length and width.

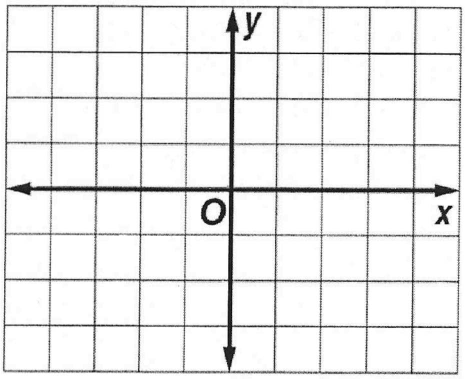
Notes 6.2 Solving Systems of Linear Equations

Solve systems of equations using:

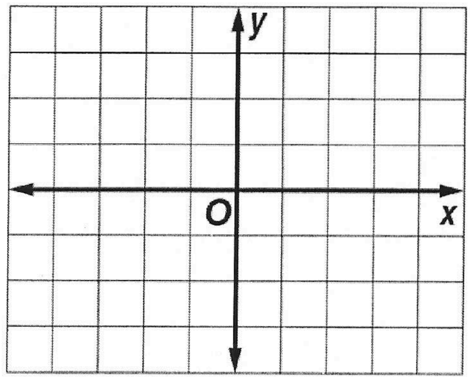
- Find the number of Solutions.
  (Infinite, One or No Solution)
- Classify the system by the # of solutions
- Modeling with linear systems.

1-6 Graph each linear system either by hand or using a graphing device. Use the graph to determine if the system has one solution, no solutions or infinitely many solutions. If there is exactly one solution, use the graph to find it.

4. $\begin{cases} 2x + 6y = 0 \\ -3x - 9y = 18 \end{cases}$



6. $\begin{cases} 12x + 15y = -18 \\ 2x + \frac{5}{2}y = -3 \end{cases}$



7-34 Solve the system, or show that it has no solution. If the system has infinitely many solutions, express them in the ordered-pair form given in Example 3.

20. $\begin{cases} 4x + 2y = 16 \\ x - 5y = 70 \end{cases}$

22. $\begin{cases} -3x + 5y = 2 \\ 9x - 15y = 6 \end{cases}$

26. $\begin{cases} 25x - 75y = 100 \\ -10x + 30y = -40 \end{cases}$

Modeling Systems of Equations:

1-

2-

3-

4-

44. **Number Problem** The sum of two numbers is twice their difference. The larger number is 6 more than twice the smaller. Find the numbers.

46. **Admission Fees** The admission fee at an amusement park is $1.50 for children and $4.00 for adults. On a certain day, 2200 people entered the park, and the admission fees collected totaled $5050. How many children and how many adults were admitted?

54. **Investments** A woman invests a total of $20,000 in two accounts, one paying 5% and the other paying 8% simple interest per year. Her annual interest is $1180. How much did she invest at each rate?

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THE SUBSTITUTION METHOD

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Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into the revised equation from Step 1 and solve.

1-8 Use the substitution method to find all solutions of the system of equations.

8. $\begin{cases} x^2 - y = 1 \\ 2x^2 + 3y = 17 \end{cases}$ $\Rightarrow y = x^2 - 1$

$2x^2 + 3(x^2 - 1) = 17$

$2x^2 + 3x^2 - 3 = 17$

$5x^2 = 20$

$x^2 = 4$

$x = \pm 2$

$y = x^2 - 1$

$y = 4 - 1$

$= 3$

$\boxed{(2,3) \text{ and } (-2,3)}$

THE ELIMINATION METHOD

Step 1 Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of its variables.

Step 2 Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

Step 3 Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

9-16 Use the elimination method to find all solutions of the system of equations.

12. $\begin{cases} (3x^2 + 4y = 17)\,2 \\ (2x^2 + 5y = 2)\,{-3} \end{cases}$

$6x^2 + 8y = 34$

$-6x^2 - 15y = -6$

$-7y = 28$

$y = -4$

$2x^2 + 5y = 2$

$2x^2 + 5(-4) = 2$

$2x^2 - 20 = 2$

$2x^2 = 22$

$x^2 = 11$

$x = \pm\sqrt{11}$

$\boxed{(\sqrt{11}, -4)\ (-\sqrt{11}, -4)}$

Solve the linear system using the substitution method:

$$2x + y = -2$$
$$5x + 3y = -8$$

Solve the linear system using the elimination method:

$$3x + 8y = -5$$
$$-2x + 2y = 18$$

17-22 Two equations and their graphs are given. Find the intersection point(s) of the graphs by solving the system.

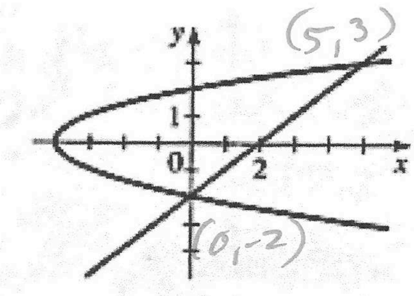
20. $\begin{cases} x - y^2 = -4 \\ x - y = 2 \end{cases}$

Check

$\boxed{0,-2}$

$0-(-2)^2=-4$ ✓

$0-(-2)=2$ ✓



$\boxed{(5,3)}$

$5-3^2=-4$ ✓

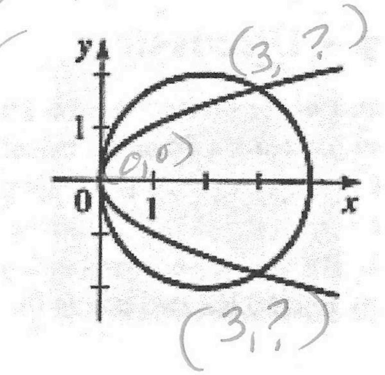
$5-3=2$ ✓

22. $\begin{cases} x^2 + y^2 = 4x \\ x = y^2 \end{cases}$

$x=y^2$

$3=y^2$

$y=\pm\sqrt{3}$



$\boxed{(0,0) \quad (3,\sqrt{3}) \quad (3,-\sqrt{3})}$

23-36 Find all solutions of the system of equations.

25. $\begin{cases} x - 2y = 2 \\ y^2 - x^2 = 2x + 4 \end{cases}$ $x = 2y+2 \Rightarrow$

$y^2-(2y+2)^2 = 2(2y+2)+4$

$y^2-(4y^2+8y+4) = 4y+4+4$

$y^2-4y^2-8y-4 = 4y+8$

$-3y^2-12y-12=0$

$y^2+4y+4=0$

$(y+2)^2=0$

$\boxed{y=-2}$

$x=2(-2)+2$

$\boxed{x=-2}$

$\boxed{(-2,-2)}$

36. $\begin{cases} \dfrac{4}{x^2} + \dfrac{6}{y^4} = \dfrac{7}{2} \\ \left(\dfrac{1}{x^2} - \dfrac{2}{y^4} = 0\right)3 \end{cases}$ $\boxed{(\pm\sqrt{2}, \pm\sqrt{2})}$

$\dfrac{3}{x^2} - \dfrac{6}{y^4} = 0$

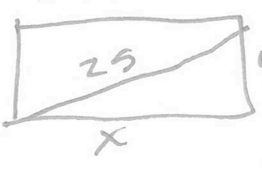
$\dfrac{7}{x^2} = \dfrac{7}{2}$

$x = \pm\sqrt{2}$

$\dfrac{1}{2} - \dfrac{2}{y^4} = 0; \quad \dfrac{1}{2} = \dfrac{2}{y^4}$

$y = \pm\sqrt{2}$

49. Dimensions of a Rectangle The perimeter of a rectangle is 70 and its diagonal is 25. Find its length and width.



$2x+2y=70$

$x^2+y^2=25$

$x^2+(35-x)^2=25$

$x+y=35$

$y=35-x$

$\boxed{15 \text{ by } 20}$

$\Rightarrow x^2+1225-70x+x^2=25$

$2x^2-70x+1200=0$

$x^2-35x+600=0$

$(x-20)(x-15)=0$

Notes 6.2 Solving Systems of Linear Equations

Solve systems of equations using:

- Find the number of Solutions.
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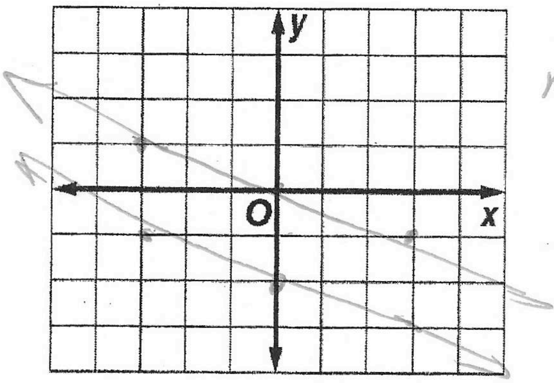
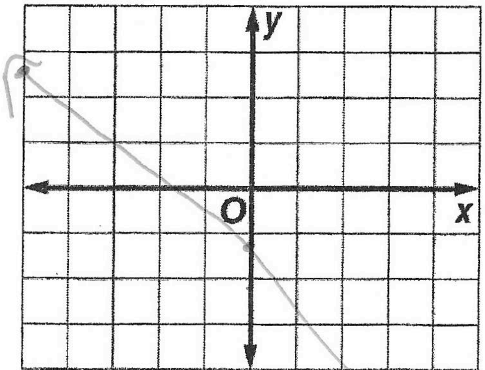
1-6 Graph each linear system either by hand or using a graphing device. Use the graph to determine if the system has one solution, no solutions or infinitely many solutions. If there is exactly one solution, use the graph to find it.

4. $\begin{cases} 2x + 6y = 0 \\ -3x - 9y = 18 \end{cases}$ $m = -\frac{1}{3}$ $b = 0$; $m = -\frac{1}{3}$ $b = -2$



no solutions

6. $\begin{cases} 12x + 15y = -18 \\ 2x + \frac{5}{2}y = -3 \end{cases}$ $m = -\frac{5}{4}$ $b = -\frac{6}{5}$; $m = -\frac{5}{4}$ $b = -\frac{6}{5}$



∞ infinite solutions

7-34 Solve the system, or show that it has no solution. If the system has infinitely many solutions, express them in the ordered-pair form given in Example 3.

20. $\begin{cases} 4x + 2y = 16 \\ x - 5y = 70 \end{cases}$ $)-4$

$-4x + 20y = -280$

$22y = -264$

$y = -12$

$x - 5(-12) = 70$

$x + 60 = 70$

$x = 10$

$\boxed{(10, -12)}$

22. $\begin{cases} (-3x + 5y = 2)3 \\ 9x - 15y = 6 \end{cases}$

$-9x + 15y = 6$

$0 = 12$

(no solution)

26. $\begin{cases} 25x - 75y = 100 \\ -10x + 30y = -40 \end{cases}$

$5x - 15y = 20$

$\Rightarrow (-x + 3y = -4)5$

$-5x + 15y = -20$

$0 = 0$

$-x + 3y = -4$ (Solve for y)

$y = \frac{1}{3}x - \frac{4}{3}$

infinite solutions

$\boxed{\left(x, \frac{1}{3}x - \frac{4}{3}\right)}$

Modeling Systems of Equations:

1- Identify the variables, usually the last question

2- Express unknown quantities as variables

3- Set up the system of equations

4- Solve the system

44. Number Problem The sum of two numbers is twice their difference. The larger number is 6 more than twice the smaller. Find the numbers.

x & y

$x+y=2(x-y) \Rightarrow x+y=2x-2y \Rightarrow 0=x-3y$

$x=2y+6$ Substitute into

$(2y+6)-3y=0$

$-y+6=0$

$\boxed{y=6}$

$x=2y+6$

$2(6)+6$

$=18$

$\boxed{18 \,\&\, 6}$

46. Admission Fees The admission fee at an amusement park is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people entered the park, and the admission fees collected totaled \$5050. How many children and how many adults were admitted?

C, A

$1.5C+4A=5050$

$(C+A=2200)-4$

$-4C-4A=-8800$

$-2.5C=-3750$

$C=1500$

1500 children
700 adults

54. Investments A woman invests a total of \$20,000 in two accounts, one paying 5% and the other paying 8% simple interest per year. Her annual interest is \$1180. How much did she invest at each rate?

x and y

14,000 at 5%
6,000 at 8%

$x+y=20{,}000 \Rightarrow x=20{,}000-y$

$.05x+.08y=1180$ ← substitute in

$.05(20{,}000-y)+.08y=1180$

$1000-.05y+.08y=1180$

→ $1000+.03y=1180$

$.03y=180$

$y=6000$