Notes 5.5 Modeling with Exponential & Logarithmic Functions

- Population Growth.
- Radioactive Decay.
- Newton's Law of Cooling.

Exponential Growth Model:

A population that experience ***exponential growth*** increases according to the model.

$$n(t) = n_0 e^{rt}$$

where: $n(t) =$ population of time t
$n_0 =$ initial size of the population
$r =$ relative rate of growth (expressed as a proportion of the population)
$t =$ time

2. Fish Population The number of a certain species of fish is modeled by the function

$$n(t) = 12e^{0.012t}$$

where t is measured in years and $n(t)$ is measured in millions.

(a) What is the relative rate of growth of the fish population? Express your answer as a percentage.
(b) What will the fish population be after 5 years?
(c) After how many years will the number of fish reach 30 million?
(d) Sketch a graph of the fish population function $n(t)$.

10. Bacteria Culture The count in a culture of bacteria was 400 after 2 hours and 25,600 after 6 hours.

(a) What is the relative rate of growth of the bacteria population? Express your answer as a percentage.
(b) What was the initial size of the culture?
(c) Find a function that models the number of bacteria $n(t)$ after t hours.
(d) Find the number of bacteria after 4.5 hours.
(e) When will the number of bacteria be 50,000?

Radioactive Decay Model:

If m_0 is the initial mass of radioactive substance with half-life h, then the mass remaining at time t is modeled by the function:

$$m(t) = m_0 e^{-rt}$$

where: $r = \frac{ln2}{h}$

16. Radioactive Thorium The mass $m(t)$ remaining after t days from a 40-g sample of thorium-234 is given by

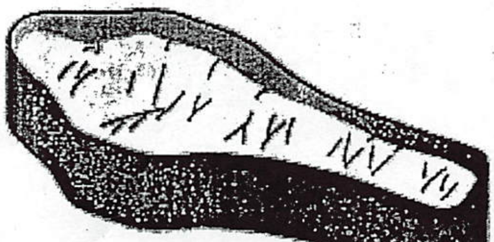
$$m(t) = 40e^{-0.0277t}$$

(a) How much of the sample will remain after 60 days?
(b) After how long will only 10 g of the sample remain?
(c) Find the half-life of thorium-234.

20. Radioactive Radon After 3 days a sample of radon-222 has decayed to 58% of its original amount.

(a) What is the half-life of radon-222?
(b) How long will it take the sample to decay to 20% of its original amount?

22. Carbon-14 Dating The burial cloth of an Egyptian mummy is estimated to contain 59% of the carbon-14 it contained originally. How long ago was the mummy buried? (The half-life of carbon-14 is 5730 years.)



Newton's Law of Cooling:
If D_0 is the initial temperature difference between an object and it's surrounding, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function:

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of the object.

24. Time of Death Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death, the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling is approximately $k = 0.1947$, assuming time is measured in hours. Suppose that the temperature of the surroundings is 60°F.

(a) Find a function $T(t)$ that models the temperature t hours after death.

(b) If the temperature of the body is now 72°F, how long ago was the time of death?

Quiz 4 Chapter 5

6. Frog Population The frog population in a small pond grows exponentially. The current population is 85 frogs, and the relative growth rate is 18% per year.

(a) Find a function that models the population after t years.

(b) Find the projected population after 3 years.

(c) Find the number of years required for the frog population to reach 600.

Notes 5.5 Modeling with Exponential & Logarithmic Functions

- Population Growth.
- Radioactive Decay.
- Newton's Law of Cooling.

Exponential Growth Model:

A population that experience ***exponential growth*** increases according to the model.

$$n(t) = n_0 e^{rt}$$

where: $n(t)$ = population of time t
n_0 = initial size of the population
r = relative rate of growth (expressed as a proportion of the population)
t = time

2. Fish Population The number of a certain species of fish is modeled by the function

$$n(t) = 12e^{0.012t}$$

where t is measured in years and $n(t)$ is measured in millions.

(a) What is the relative rate of growth of the fish population? Express your answer as a percentage. 1.2%

(b) What will the fish population be after 5 years? → $12e^{.012(5)} \approx 12.74$

(c) After how many years will the number of fish reach 30 million? → $30 = 12e^{.012t}$ $t \approx 76.4$ yrs.

~~(d)~~ Sketch a graph of the fish population function $n(t)$.

10. Bacteria Culture The count in a culture of bacteria was 400 after 2 hours and 25,600 after 6 hours.

(a) What is the relative rate of growth of the bacteria population? Express your answer as a percentage. 1.0397

(b) What was the initial size of the culture? 50

(c) Find a function that models the number of bacteria $n(t)$ after t hours.

(d) Find the number of bacteria after 4.5 hours.

(e) When will the number of bacteria be 50,000?

$(2, 400)$

$400 = n_0 e^{r(2)}$

$(6, 25{,}600)$

$25{,}600 = n_0 e^{r(6)}$

Divide one equation by other to solve for r

$$\frac{25{,}000}{400} = \frac{n_0 e^{r(6)}}{n_0 e^{r(2)}}$$

$64 = e^{4r}$ $r \approx 1.0397$

b) $400 = n_0 e^{1.0397(2)}$

$n_0 = 50$

c) $n(t) = 50e^{1.0397t}$

d) $n(t) = 50e^{1.0397(4.5)} \approx 5388$

e) $50{,}000 = 50e^{1.0397t}$

$t \approx 6.6$

Radioactive Decay Model:
If m_0 is the initial mass of radioactive substance with half-life h, then the mass remaining at time t is modeled by the function:

$$m(t) = m_0 e^{-rt}$$

where: $r = \frac{ln2}{h}$

16. Radioactive Thorium The mass $m(t)$ remaining after t days from a 40-g sample of thorium-234 is given by

$$m(t) = 40e^{-0.0277t}$$

(a) How much of the sample will remain after 60 days?
(b) After how long will only 10 g of the sample remain?
(c) Find the half-life of thorium-234.

a) $40e^{-.0277(60)} \approx 7.6$ g.

b) $10 = 40e^{-.0277(t)}$ $t \approx 50$

c) $\frac{1}{2} = e^{-.0277t}$

$t \approx 25$ years

20. Radioactive Radon After 3 days a sample of radon-222 has decayed to 58% of its original amount.

$.58 = e^{r(3)}$
$r \approx -0.18$

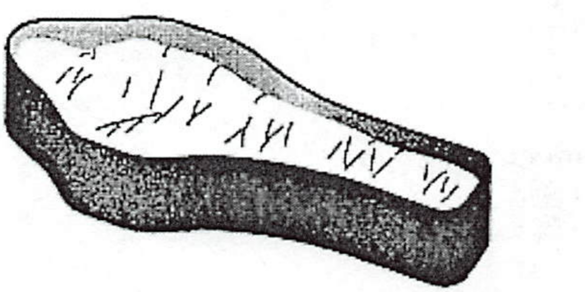
(a) What is the half-life of radon-222?
(b) How long will it take the sample to decay to 20% of its original amount?

a) $\frac{1}{2} = e^{-.18t}$ $t \approx 3.85$ days

b) $.2 = e^{-.18t}$ $t \approx 8.9$ days

22. Carbon-14 Dating The burial cloth of an Egyptian mummy is estimated to contain 59% of the carbon-14 it contained originally. How long ago was the mummy buried? (The half-life of carbon-14 is 5730 years.)

$r = \frac{ln 2}{h}$ $r = \frac{ln 2}{5730} \approx -.000121$



$.59 = e^{-.000121t}$

$t \approx 4360$ years

Newton's Law of Cooling:
If D_0 is the initial temperature difference between an object and it's surrounding, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function:

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of the object.

$D_0 = T_i - T_s$

T_i = initial temp

T_s = Surrounding temp

24. Time of Death Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6°F. Immediately following death, the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling is approximately $k = 0.1947$, assuming time is measured in hours. Suppose that the temperature of the surroundings is 60°F.

(a) Find a function $T(t)$ that models the temperature t hours after death.

(b) If the temperature of the body is now 72°F, how long ago was the time of death?

$$T(t) = T_s + D_0 e^{-kt}$$

a) $T(t) = 60 + 38.6e^{-.1947t}$

b) $72 = 60 + 38.6e^{-.1947t}$

$t \approx 6$ hours.