

Notes 5.4b Solving Logarithmic Equations

- Use exponents to solve logarithmic equations.
- Use logarithms to solve compound interest for time required to grow an investment.

Guidelines for solving logarithmic equations:

- 1- Isolate the logarithmic term on one side of the equation; you may first need to combine the logarithmic terms.
- 2- Write the equation in exponential form (or raise the base to each side of the equation).
- 3- Solve for the variable.

Property of equality for logarithmic equations:

If $\log_b x = \log_b y$ then _____

Solve $\log_7(6x - 16) = \log_7(x - 1)$

Exponentiate each side of an equation.

Solve $\log_5(3x - 8) = 2$

Check for extraneous solutions:

Solve $\log 5x + \log(x - 1) = 2$

35-50 Solve the Logarithmic Equation for x.

36. $\ln(2 + x) = 1$

38. $\log(x - 4) = 3$

35-50 Solve the Logarithmic Equation for x .

44. $2 \log x = \log 2 + \log(3x - 4)$

48. $\log x + \log(x - 3) = 1$

78. **Transparency of a Lake** Environmental scientists measure the intensity of light at various depths in a lake to find the “transparency” of the water. Certain levels of transparency are required for the biodiversity of the submerged macrophyte population. In a certain lake the intensity of light at depth x is given by

$$I = 10e^{-0.008x}$$

where I is measured in lumens and x in feet.

(a) Find the intensity I at a depth of 30 ft.

(b) At what depth has the light intensity dropped to $I = 5$?

5. $\ln(7x - 13)$
 $= \ln(2x + 17)$

6. $\log_3(2x + 9) = 3$

7. $\log_4(10x + 624) = 5$

8. $\log_6(x - 9) + \log_6 x$
 $= 2$

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Property of equality for logarithmic equations:

If $\log_b x = \log_b y$ then $x = y$

Solve $\log_7(6x - 16) = \log_7(x - 1)$

$$\begin{aligned} 6x - 16 &= x - 1 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

$$\begin{array}{l} 6x - 16 \qquad x - 1 \\ 6(3) - 16 \qquad 3 - 1 \\ 2 \quad \checkmark \qquad 2 \quad \checkmark \end{array}$$

Exponentiate each side of an equation.

Solve $\log_5(3x - 8) = 2$

or $5^{\log_5(3x-8)} = 5^2$

$$\begin{aligned} 3x - 8 &= 25 \\ 3x &= 33 \\ \boxed{x = 11} \end{aligned}$$

$\log_5 25 = 2 \checkmark$

$$\underline{5^2 = 3x - 8}$$

$$\log_5(3(11) - 8) = 2$$

Check for extraneous solutions:

Solve $\log 5x + \log(x - 1) = 2$

$$\begin{aligned} \log_5 5x(x-1) &= 2 \\ 5x(x-1) &= 10^2 \\ 5x^2 - 5x &= 100 \end{aligned}$$

$$\begin{aligned} 5x^2 - 5x - 100 &= 0 \\ x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \end{aligned}$$

$$\boxed{x = 5} \text{ or } \boxed{x = -4}$$

35-50 Solve the Logarithmic Equation for x.

36. $\ln(2 + x) = 1$

$$\begin{aligned} 2 + x &= e^1 \\ x &= e^1 - 2 \approx .718 \end{aligned}$$

38. $\log(x - 4) = 3$

$$\begin{aligned} x - 4 &= 10^3 \\ \boxed{x = 1004} \end{aligned}$$

44. $2 \log x = \log 2 + \log(3x - 4)$

$$\log x^2 = \log 2(3x-4)$$

$$x^2 = 6x - 8$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 2 \text{ or } 4$$

48. $\log x + \log(x - 3) = 1$

$$\log x(x-3) = 1$$

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5 \text{ or } x = -2$$

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$$I = 10e^{-0.008x}$$

where I is measured in lumens and x in feet.

(a) Find the intensity I at a depth of 30 ft.

$$I = 10e^{-0.008(30)} \approx 7.87 \text{ lumens}$$

(b) At what depth has the light intensity dropped to $I = 5$?

$$5 = 10e^{-0.008x}$$

86.6 feet

5. $\ln(7x - 13) = \ln(2x + 17)$

$$7x - 13 = 2x + 17$$

$$5x = 30$$

$$x = 6$$

6. $\log_3(2x + 9) = 3$

$$2x + 9 = 3^3$$

$$2x + 9 = 27$$

$$2x = 18$$

$$x = 9$$

7. $\log_4(10x + 624) = 5$

$$10x + 624 = 4^5$$

$$10x + 624 = 1024$$

$$10x = 400$$

$$x = 40$$

8. $\log_6(x - 9) + \log_6 x = 2$

$$\log_6(x-9)x = 2$$

$$x^2 - 9x = 6^2$$

$$x^2 - 9x - 36 = 0$$

$$(x-12)(x+3) = 0$$

$$x = 12 \text{ or } x = -3$$