Notes 5.4A
Modeling with Exponential and Logarithmic Functions

- Use logarithms to solve exponential equations.
- Solve exponential equations graphically.
- Use logarithms to solve compound interest for time required to grow an investment.

Solve by equating exponents:

$b^x = b^y$ then _____

$3^4 = 9^y$ $2^y = 8^3$ $9^y = 27^2$

Solve
$64^x = 16^{x+1}$

Guidelines for solving exponential equations:

1-

2-

3-

Take a logarithm of each side:
Solve: $6^x = 27$

1-26 Find the solution of the exponential equation, correct to four decimal places.

4. $e^{-2x} = \frac{1}{10}$ 6. $3^{2x-1} = 5$ 8. $2e^{12x} = 17$

10. $4\left(1+10^{5x}\right) = 9$ 16. $e^{3-5x} = 16$ 21. $2^{3x+1} = 3^{x-2}$

Solve:
$6e^{0.25x} + 8 = 20$

27 – 34 Solve the equation:

28. $x^2 10^x - x10^x = 2(10^x)$

32. $e^{2x} - e^x - 6 = 0$

55-62 Use a graphing device to find all solutions of the equation, correct to two decimal places.

59. $e^x = -x$

60. $2^{-x} = x - 1$

$A = Pe^{rt}$

68. **Compound Interest.** A man invests \$6500 in an account that pays 6% interest per year, compounded continuously.

a) What is the amount after 2 years?

b) How long will it take for the amount to be \$8000?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

70. **Compound Interest.** Nancy wants to invest \$4000 in savings certificates that bear an interest rate of 9.75% per year, compounded semi-annually. How long a time period should she choose in order to save an amount of \$5000.

Notes 5.4A
Modeling with Exponential and Logarithmic Functions

- Use logarithms to solve exponential equations.
- Solve exponential equations graphically.
- Use logarithms to solve compound interest for time required to grow an investment.

Solve by equating exponents:

$b^x = b^y$ then $x = y$

$3^4 = 9^y$ $\quad 3^4 = 3^{2y}$ $\quad \boxed{y = 2}$

$2^y = 8^3$ $\quad 2^y = (2^3)^3$ $\quad \boxed{y = 9}$

$9^y = 27^2$ $\quad 3^{2y} = 3^{3\cdot 2}$ $\quad 2y = 6$ $\quad \boxed{y = 3}$

Solve
$64^x = 16^{x+1}$

$(4^3)^x = (4^2)^{x+1}$
$4^{3x} = 4^{2x+2}$
$3x = 2x + 2$ $\quad \boxed{x = 2}$ $\quad 64^2 = 16^3$

Guidelines for solving exponential equations:

1- Isolate the exponential expression on one side of the equation.

2- Take the logarithm of each side, and then use the Law of Logarithms to "bring down the exponent."

3- Solve for the variable

Take a logarithm of each side:

Solve: $6^x = 27$

$\log_6 6^x = \log_6 27 \Rightarrow x \log_6 6 = \log_6 27$
$x \cdot 1 = x = \frac{\log 27}{\log 6} \approx 1.84$

1-26 Find the solution of the exponential equation, correct to four decimal places.

4. $e^{-2x} = \frac{1}{10}$

$\ln e^{-2x} = \ln \frac{1}{10}$ $\quad \boxed{\approx 1.15}$
$-2x = \ln \frac{1}{10}$ $\quad x = \frac{\ln \frac{1}{10}}{-2}$

6. $3^{2x-1} = 5$

$\log_3 3^{2x-1} = \log_3 5$
$2x - 1 = \frac{\log 5}{\log 3}$
$\boxed{x \approx 1.23}$

8. $2e^{12x} = 17$

$e^{12x} = \frac{17}{2}$
$12x = \ln \frac{17}{2}$
$\boxed{x \approx 0.178}$

10. $4(1 + 10^{5x}) = 9$

$1 + 10^{5x} = 2.25$
$10^{5x} = 1.25$
$5x = \log_{10} 1.25$
$\boxed{x \approx .019}$

16. $e^{3-5x} = 16$

$3 - 5x = \ln 16$
$\boxed{x \approx 0.45}$

21. $2^{3x+1} = 3^{x-2}$

$\log_2 2^{3x+1} = \log_2 3^{x-2}$
$3x + 1 = (x - 2)\log_2 3$
$(x - 2)1.585$
$3x + 1 = 1.585x - 3.17$
$1.415x = -4.17$ $\quad \boxed{x \approx -2.94}$

Solve:
$6e^{0.25x} + 8 = 20$

$6e^{0.25x} = 12$
$e^{0.25x} = 2$ $\quad 0.25x = \ln 2$ $\quad \boxed{x = 2.77}$

27 – 34 Solve the equation:

28. $x^2 10^x - x10^x = 2(10^x)$ $x^2 10^x - x10^x - 2(10^x) = 0$

$10^x \neq 0$ $10^x(x^2 - x - 2) = 0$ $x = 2, x = -1$

$10^x(x-2)(x+1) = 0$

32) $e^{2x} - e^x - 6 = 0$

$(e^x + 2)(e^x - 3) = 0$

$e^x = -2$ $e^x = 3$

$e^x = -2$ no sol. $e^x = 3$ $x = \ln 3 \approx 1.099$

55-62 Use a graphing device to find all solutions of the equation, correct to two decimal places.

59. $e^x = -x$ $e^x + x = 0$ $(-0.567, 0)$

60. $2^{-x} = x - 1$ $2^{-x} - x + 1 = 0$ $x \approx 1.383$

$A = Pe^{rt}$

68. **Compound Interest.** A man invests \$6500 in an account that pays 6% interest per year, compounded continuously.

a) What is the amount after 2 years? $6500e^{.06(2)} = \$7328.73$

b) How long will it take for the amount to be \$8000?

$8{,}000 = 6500e^{.06t}$

$\frac{8000}{6500} = \frac{16}{13} = e^{.06t}$

$\ln \frac{16}{13} = .06t$

$t \approx 3.5$ years

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

70. **Compound Interest.** Nancy wants to invest \$4000 in savings certificates that bear an interest rate of 9.75% per year, compounded semi-annually. How long a time period should she choose in order to save an amount of \$5000.

$5{,}000 = 4{,}000\left(1 + \frac{.0975}{2}\right)^{2t}$

$1.25 = (1.04875)^{2t}$

$\log_{1.04875} 1.25 = 2t$

2.3 years