Notes 5.3
De Moivre's Theorem

Use De Moivre's Theorem to find the power of a complex number
Use De Moivre's Theorem for finding roots

Use De Moivre's Theorem to find the power of a complex number:

De Moivre's Theorem:
Let $z = r(\cos\theta + isin\theta)$ and let n be a positive integer.
Then:

$z^n =$ $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

$= r^n(\text{cis } n\theta)$

$r = \sqrt{2} \quad \theta = 315^\circ$

In Exercises 1 to 14, find the indicated power. Write the answer in standard form.

3. $[2(\cos 240° + i\sin 240°)]^5$

$2^5(\cos(5\cdot 240) + i\sin(5\cdot 240)]$

$32(\cos(1200°) + i\sin(1200°)]$

$(1200° = 120°)$

$32(\cos 120° + i\sin 120°)$

$32(-1/2 + i(\frac{\sqrt{3}}{2})) = \boxed{-16 + 16i\sqrt{3}}$

9. $(1 - i)^{10}$

$(1-i) = \boxed{\sqrt{2} \text{ cis } 315°}$

$(\sqrt{2} \text{ cis } 315°)^{10}$

$= (\sqrt{2})^{10} \text{ cis}(10\cdot 315)$

$= 32 \text{ cis } 3150$ $\quad$ $3150° = 270°$

$32 \text{ cis } 270°$

$32(0 + i(-1))$

$= \boxed{-32i}$

Use De Moivre's Theorem for finding roots:

Finding nth roots of a complex number:
if $z = r(\cos\theta + isin\theta)$, then the n distinct complex numbers.

$$\sqrt[n]{r}\left(\cos\frac{\theta + 360°\cdot k}{n} + i\sin\frac{\theta + 360°\cdot k}{n}\right)$$

$$\text{or} \quad \sqrt[n]{r}\left(\text{cis}\frac{\theta + 360°\cdot k}{n}\right)$$

$$\text{or} \quad r^{1/n}\left(\text{cis}\frac{\theta + 360°\cdot k}{n}\right)$$

where k = 0, 1, 2, ..., $n - 1$, are the *nth* roots of the complex number z.

In Exercises 15 to 28, find all of the indicated roots. Write all answers in $\boxed{\text{standard form.}}$

19. The five fifth roots of -1

Change -1 to trig form $\boxed{\text{cis } 180°}$

cis 180°

$n = 5$

$r = 1$

$\theta = 180°$

Finding 5 roots so use $k = 0, 1, 2, 3, 4$

$\boxed{k=0}$ $\sqrt[5]{1} \text{ cis } \frac{180 + 360 \cdot (0)}{5} = 1 \text{ cis } 36 \approx \boxed{.809 + .588i}$ *

$k=1$ $\sqrt[5]{1} \text{ cis } \frac{180 + 360(1)}{5} = 1 \text{ cis } 108 \approx \boxed{-.309 + .951i}$ *

$k=2$ $\sqrt[5]{1} \text{ cis } \frac{180 + 360(2)}{5} = 1 \text{ cis } 180 = \boxed{-1}$ *

$k=3$ $\sqrt[5]{1} \text{ cis } \frac{180 + 360(3)}{5} = 1 \text{ cis } 252 = \boxed{-.309 - .951i}$ *

$k=4$ $\sqrt[5]{1} \text{ cis } \frac{180 + 360(4)}{5} = 1 \text{ cis } 324 \approx \boxed{.809 - .588i}$ *

23. The four fourth roots of $1 + i$

$1 + i = \sqrt{2} \text{ cis } 45°$

$k = 0$ $\sqrt[4]{\sqrt{2}} = 2^{1/8} \text{ cis } \frac{45 + 360 \cdot 0}{4}$

$= 2^{1/8} \text{ cis } 11.25 = \boxed{1.070 + .213i}$ *

$k=1$ $2^{1/8} \text{ cis } \frac{45 + 360 \cdot 1}{4} = 2^{1/8} \text{ cis } 101.25 = \boxed{-.213 + 1.070i}$ *

$k=2$ $2^{1/8} \text{ cis } \frac{45 + 360 \cdot 2}{4} = 2^{1/8} \text{ cis } 191.25 = \boxed{-1.070 - 2.13i}$ *

$k=3$ $2^{1/8} \text{ cis } \frac{45 + 360 \cdot 3}{4} = 2^{1/8} \text{ cis } 281.25 = \boxed{.213 - 1.070i}$ *

In Exercises 29 to 40, find all roots of the equation. Write the answers in trigonometric form.

30. $x^5 - 32 = 0$ $x^5 = 32$ $x = \sqrt[5]{32}$

Change 32 to trig form: 32 cis 0°

$k=0$ $\sqrt[5]{32} \text{ cis } \frac{0° + 360 \cdot 0}{5} = 2 \text{ cis } 0 = \boxed{2}$

$k=1$ $\sqrt[5]{32} \text{ cis } \frac{0° + 360° \cdot 1}{5} = 2 \text{ cis } 72° = \boxed{.618 + 1.902i}$

$k=2$ $\sqrt[5]{32} \text{ cis } \frac{0° + 360° \cdot 2}{5} = 2 \text{ cis } 144° = \boxed{-1.618 + 1.176i}$

$k=3$ $\sqrt[5]{32} \text{ cis } \frac{0 + 360 \cdot 3}{5} = 2 \text{ cis } 216 = \boxed{-1.618 - 1.176i}$

$k=4$ $\sqrt[5]{32} \text{ cis } \frac{0 + 360 \cdot 4}{5} = 2 \text{ cis } 288 = \boxed{.618 - 1.902i}$

All n^{th} roots of z are equally spaced on a circle.

notice each root increases by 72 for #30 and $\frac{360}{5} = 72°$