

Notes 5.2 Logarithmic Functions

Use definition of Logarithms to switch from Logarithm to Exponential & Exponential to Logarithm

**Definition of logarithms with base  $b$ .**

$$\log_b y = \text{---} \text{ if and only if } b^x =$$

1-6 Express the equation in exponential form.

3. (a)  $\log_8 2 = \frac{1}{3}$  (b)  $\log_2 \left(\frac{1}{8}\right) = -3$

6. (a)  $\ln(x+1) = 2$  (b)  $\ln(x-1) = 4$

7- 12 Express the equation in logarithmic form.

10. (a)  $4^{-3/2} = 0.125$  (b)  $7^3 = 343$

13-22 Evaluate the expression.

14. (a)  $\log_5 5^4$  (b)  $\log_4 64$  (c)  $\log_9 9$

18. (a)  $\log_5 125$  (b)  $\log_{49} 7$  (c)  $\log_9 \sqrt{3}$

**Solve Logarithmic equations**

23-30 Use the definition of the logarithmic function to find  $x$ .

23. (a)  $\log_2 x = 5$  (b)  $\log_2 16 = x$

30. (a)  $\log_x 6 = \frac{1}{2}$  (b)  $\log_x 3 = \frac{1}{3}$

31-34 Use a calculator to evaluate the expression, correct to four decimal places.

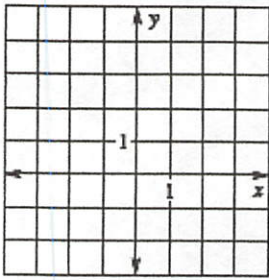
32. (a)  $\log 50$  (b)  $\log \sqrt{2}$  (c)  $\log(3\sqrt{2})$

34. (a)  $\ln 27$  (b)  $\ln 7.39$  (c)  $\ln 54.6$

## Graph Logarithms

Parent graphs for logarithmic functions:

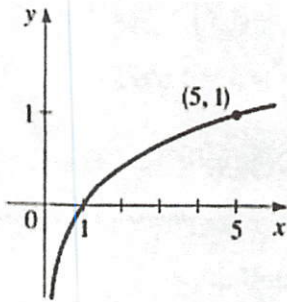
$$y = \log_b x$$



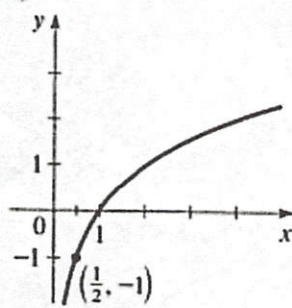
Vertical Asymptote: \_\_\_\_\_ Domain: \_\_\_\_\_ Range: \_\_\_\_\_

35-38 Find the function of the form  $y = \log_a x$  whose graph is given.

35.



36.



39-44 Match the logarithmic function with one of the graphs labeled I-VI

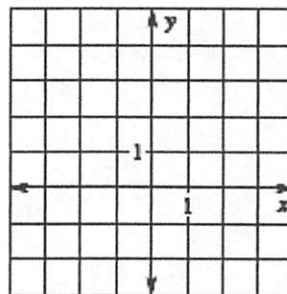
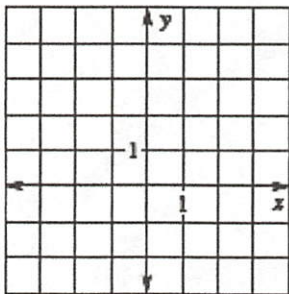
41.  $f(x) = 2 + \ln x$

41.  $f(x) = \ln(-x)$

47-56 Graph the function, not by plotting points, but by starting from the graphs in Figures 4 and 9. State the domain, range, and asymptote.

51.  $y = 2 + \log_3 x$

52.  $y = \log_3(x - 1) - 2$



57-62 Find the Domain of the function:

58.  $f(x) = \log_5(8 - 2x)$

60.  $g(x) = \ln(x - x^2)$

79. Investment The time required to double the amount of an investment at an interest rate  $r$  compounded continuously is given by:

$$t = \frac{\ln 2}{r}$$

Find the time required to double an investment at 6%, 7%, and 8%.

## Use definition of Logarithms to switch from Logarithm to Exponential &amp; Exponential to Logarithm

Definition of logarithms with base  $b$ .

$$\log_b y = x \text{ if and only if } b^x = y$$

1-6 Express the equation in exponential form.

3. (a)  $\log_8 2 = \frac{1}{3}$   $8^{1/3} = 2$  (b)  $\log_2 \left(\frac{1}{8}\right) = -3$

6. (a)  $\ln(x+1) = 2$  (b)  $\ln(x-1) = 4$

$$e^2 = (x+1)$$

7-12 Express the equation in logarithmic form.

10. (a)  $4^{-3/2} = 0.125$  (b)  $7^3 = 343$

$$\log_4 0.125 = -\frac{3}{2}$$

13-22 Evaluate the expression.

14. (a)  $\log_5 5^4 = 4$  (b)  $\log_4 64$  (c)  $\log_9 9$

$$5^y = 5^4 \quad | \quad y = 4$$

18. (a)  $\log_5 125 = 3$  (b)  $\log_{49} 7$  (c)  $\log_9 \sqrt{3}$

$$5^y = 125 \quad | \quad y = 3$$

## Solve Logarithmic equations

23-30 Use the definition of the logarithmic function to find  $x$ .

23. (a)  $\log_2 x = 5$  (b)  $\log_2 16 = x$

$$2^5 = x \quad | \quad x = 32$$

30. (a)  $\log_x 6 = \frac{1}{2}$  (b)  $\log_x 3 = \frac{1}{3}$

$$x^{1/2} = 6 \quad | \quad x = 36$$

31-34 Use a calculator to evaluate the expression, correct to four decimal places.

32. (a)  $\log 50$  (b)  $\log \sqrt{2}$  (c)  $\log(3\sqrt{2})$

$$1.6990$$

$$0.1505$$

34. (a)  $\ln 27$  (b)  $\ln 7.39$  (c)  $\ln 54.6$

$$3.2958$$

$$2.0001$$

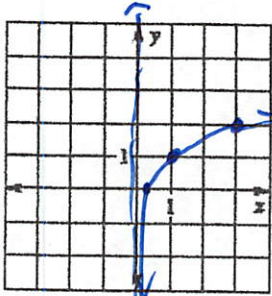


# Graph Logarithms

Parent graphs for logarithmic functions:

$$y = \log_b x \rightarrow b^y = x$$

graph the exponential form



$$y = \log_3 x$$

$$\rightarrow 3^y = x$$

choose values for y instead of x

x	y
1/3	-1
1	0
3	1

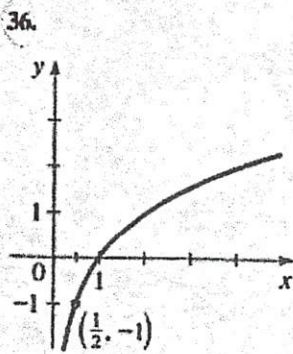
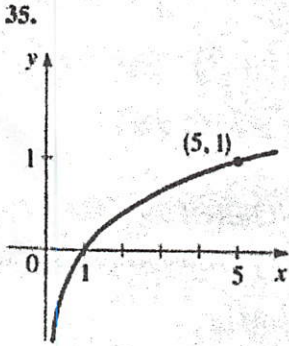
[logs and exponential's are inverse functions]

Vertical Asymptote:  $x = 0$

Domain:  $(0, \infty)$

Range:  $\mathbb{R}$

35-38 Find the function of the form  $y = \log_a x$  whose graph is given.



35)

$$b^y = x$$

$$b^{-1} = 5$$

$$b = 5$$

$$\log_5 x = y$$

36)

$$b^y = x$$

$$b^{-1} = 1/2$$

$$b = 2$$

$$\log_2 x = y$$

39-44 Match the logarithmic function with one of the graphs labeled I-VI

41.  $f(x) = 2 + \ln x$

III

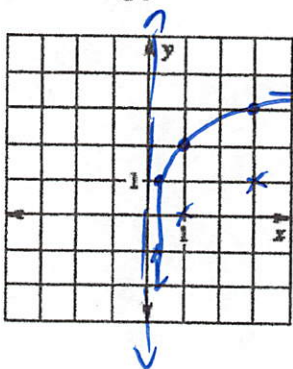
42.  $f(x) = \ln(-x)$

IV

47-56 Graph the function, not by plotting points, but by starting from the graphs in Figures 4 and 9.

State the domain, range, and asymptote.

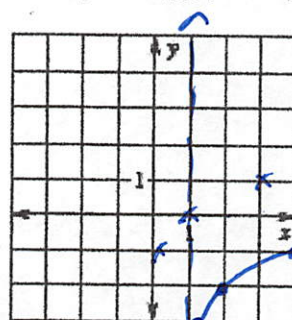
51.  $y = 2 + \log_3 x$



Shift up 2

D:  $(0, \infty)$   
R:  $\mathbb{R}$   
Asy:  $x = 0$

52.  $y = \log_3(x - 1) - 2$



use graph above  $y = \log_3 x$

Shift right 1 down 2

D:  $(1, \infty)$   
R:  $\mathbb{R}$   
Asy:  $x = 1$

57-62 Find the Domain of the function:

58.  $f(x) = \log_5(8 - 2x)$

$$8 - 2x > 0$$

$$-2x > -8$$

$$x < 4 \quad (4, \infty)$$

60.  $g(x) = \ln(x - x^2)$

$$x - x^2 > 0$$

$$x(x - 1) > 0$$

find critical points



79. Investment The time required to double the amount of an investment at an interest rate  $r$  compounded continuously is given by:

$$t = \frac{\ln 2}{r}$$

Find the time required to double an investment at 6%, 7%, and 8%.

$$\frac{\ln 2}{.06} = 11.6 \text{ years}$$

$$\frac{\ln 2}{.07} = 9.9 \text{ years}$$

Test a point (2)?  $2 - 2^2 > 0$   
so solution  $(0, 1)$