Notes 4.7A Roots and Zeros:
I understand how zeros, roots, factors and intercepts are related.
I can find the zeros of a polynomial using the quadratic equation.
I can find a polynomial when given the zeros, intercepts, factors, or roots.

I understand how zeros, roots, factors and intercepts are related.
The rational zero theorem: Every rational zero has the form:

$\frac{p}{q} =$ factors of constant / factors of leading coefficient

I can find the zeros of a polynomial using the quadratic equation.

Quadratic Equation: $ax^2+bx+c=0$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solve using the quadratic equation:

$f(x) = x^2 - 4x - 1$

$\frac{4 \pm \sqrt{16-4}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$

$2 \pm \sqrt{3}$

$f(x) = 2x^2 - 3x - 1$

$\frac{3 \pm \sqrt{9-4(2)(-1)}}{2(2)} = \frac{3 \pm \sqrt{17}}{4}$

You try ☺
$f(x) = x^2 - 2x - 2$

$\frac{2 \pm \sqrt{4-4(1)(-2)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

Given a polynomial and one of its zeros, find the remaining zeros (Use the quadratic equation).

$f(x) = x^3 - 2x^2 - 12x - 8;\ -2$

-2 | 1 −2 −12 −8 / −2 8 8 / 1 −4 −4 0

$x^2 - 4x - 4$

$\frac{4 \pm \sqrt{16-4(1)(-4)}}{2(1)} = \frac{4 \pm \sqrt{32}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$

$f(x) = x^3 - 8x^2 + 15x + 4;\ 4$

4 | 1 −8 15 4 / 4 −16 −4 / 1 −4 −1 0

$\frac{4 \pm \sqrt{16-4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$f(x) = 2x^3 - 3x^2 - 5x - \frac{3}{2};\ -\frac{1}{2}$

$-\frac{1}{2}$ | 2 −3 −5 $-\frac{3}{2}$ / −1 2 $\frac{3}{2}$ / 2 −4 −3 0

$\frac{4 \pm \sqrt{16-4(2)(-3)}}{2(2)} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$

$f(x) = x^4 + x^3 - 5x^2 - 4x + 4;\ 2$

2 | 1 1 −5 −4 4 / 2 6 2 −4 / 1 3 1 −2 0

-2 | 1 3 1 −2 / −2 −2 2 / 1 1 −1 0

$\frac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$

You try ☺
$f(x) = x^3 - 4x^2 + 2x + 1;\ 1$

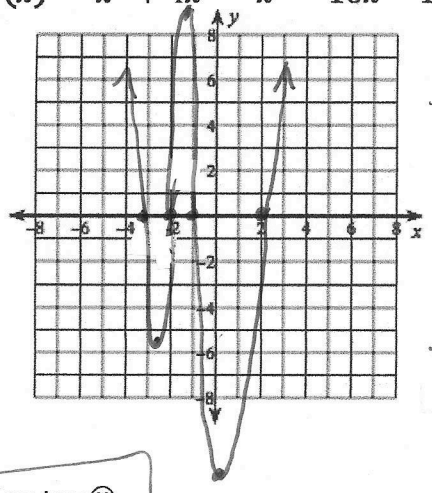
1 | 1 −4 2 1 / 1 −3 −1 / 1 −3 −1 0

$\frac{3 \pm \sqrt{9-4(1)(-1)}}{2} = \frac{3 \pm \sqrt{13}}{2}$

(Finding zeros on next page)
& test points

Identify the zeros, factors, and x-intercepts of each function. Then graph:

$f(x) = x^4 + 4x^3 - x^2 - 16x - 12$



Zeros
$-3, -2, -1, 2$

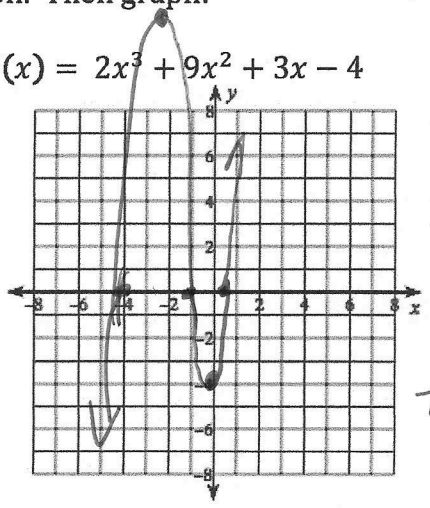
End Behavior
↑ ↑

y-int $(0, -12)$

test points

x	y
-1.5	5.8
-2.5	10.8

$f(x) = 2x^3 + 9x^2 + 3x - 4$



Zeros
$-4, -1, \frac{1}{2}$

End Behavior
↓ ↑

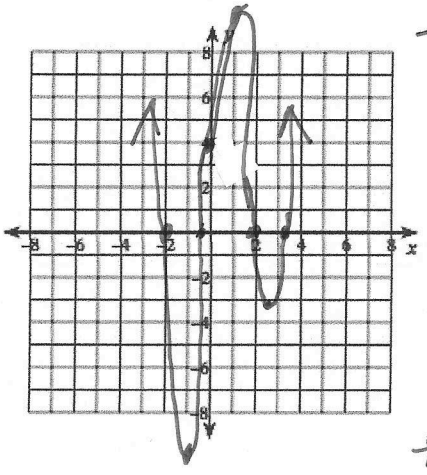
y-int $(0, -4)$

Test points

x	y
-3	14

You try ☺

$f(x) = x^4 - 3x^3 - 5x^2 + 12x + 4$



Zeros
$2, -2, 3.3, -0.3$

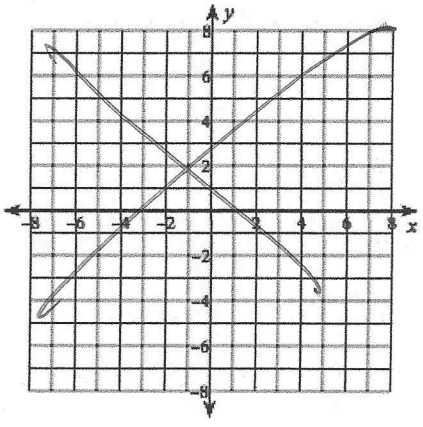
End Behavior
↑ ↑

y-int
$(0, 4)$

test points

x	y
1	9
2.5	-5.1
-1	-9

~~$f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$~~



I can find a polynomial when given the zeros, intercepts, factors, or roots.

c (c,0) (x-c)

Write a polynomial of least degree with integral coefficients that has the given zeros.

-1, 2, 3

$(x+1)(x-2)(x-3)$
$(x^2-x-2)(x-3)$
$x^3-3x^2-x^2+3x-2x+6$
x^3-4x^2+x+6

-2, 2, 4

$(x+2)(x-2)(x-4)$
$(x^2-4)(x-4)$
$x^3-4x^2-4x+16$

$-3, 1, \sqrt{2}, -\sqrt{2}$

$(x+3)(x-1)(x-\sqrt{2})(x+\sqrt{2})$
$(x^2+2x-3)(x^2-2)$
$x^4-2x^2+2x^3-4x-3x^2+6$
$x^4+2x^3-5x^2-4x+6$

You try ☺

-3, 1, 2

$(x+3)(x-1)(x-2)$
$(x+3)(x^2-3x+2)$
x^3-3x^2+2x
$3x^2-9x+6$
x^3-7x+6

$4, \sqrt{5}, -\sqrt{5}$

$(x-4)(x-\sqrt{5})(x+\sqrt{5})$
$(x-4)(x^2-5)$
$x^3-4x^2-5x+20$

Work to find the zero's on other page

Identify the zeros, factors, and x-intercepts of each function. Then graph:

$f(x) = x^4 + 4x^3 - x^2 - 16x - 12$

-1 | 1 4 −1 −16 −12

−1 −3 4 12

2 | 1 3 −4 −12 0

2 10 12

1 5 6 | 0

Zeros: −1, 2, −3, 2

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

test point use remander theorem

−1.5 | 1 4 1 −16 −12

−1.5 −3.75 4.125 17.8125

1 2.5 −2.75 −11.875 5.8

You try ☺

$f(x) = x^4 - 3x^3 - 5x^2 + 12x + 4$

test point

1 | 9

1 | 1 −3 −5 12 4

1 −2 −7 5 9

2 | 1 −3 −5 12 4

2 −2 −14 −4

−2 | 1 −1 −7 −2 0

−2 6 2

1 −3 −1 0

$x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2(1)}$

$= \frac{3 \pm \sqrt{13}}{2}$

$\frac{3 - \sqrt{13}}{2} = -0.3$

$\frac{3 + \sqrt{13}}{2} = 3.3$

$f(x) = 2x^3 + 9x^2 + 3x - 4$

−1 | 2 9 3 −4

−2 −7 4

2 7 −4 0

$2x^2 + 7x - 4 = 0$

$(2x-1)(x+4) = 0$

Zeros: $-1, \frac{1}{2}, -4$

y-int (0, −4)

test points

−3 | 2 9 3 −4

−6 −9 18

2 3 −6 14

test points

−1 | 1 −3 −5 12 4

−1 4 1 −13

1 −4 −1 13 −9

2.5 | 1 −3 −5 12 4

2.5 −1.25 −15.625 −9.062

1 −0.5 −6.25 −3.625 −5.062