

Notes 4.7A Roots and Zeros:

- I understand how zeros, roots, factors and intercepts are related.
- I can find the zeros of a polynomial using the quadratic equation.
- I can find a polynomial when given the zeros, intercepts, factors, or roots.

I understand how zeros, roots, factors and intercepts are related.

The rational zero theorem: Every rational zero has the form:

$$\frac{p}{q} = \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

I can find the zeros of a polynomial using the quadratic equation.

Quadratic Equation:

Quadratic Formula:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve using the quadratic equation:

$$f(x) = x^2 - 4x - 1 \quad \frac{4 \pm \sqrt{16 - 4}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$$f(x) = 2x^2 - 3x - 1 \quad \frac{3 \pm \sqrt{9 - 4(2)(-1)}}{2(2)} = \frac{3 \pm \sqrt{17}}{4}$$

You try ☺

$$f(x) = x^2 - 2x - 2$$

$$\frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

Given a polynomial and one of its zeros, find the remaining zeros (Use the quadratic equation).

$$f(x) = x^3 - 2x^2 - 12x - 8; -2$$

$$\begin{array}{r} -2 \downarrow \\ 1 \quad -2 \quad -12 \quad -8 \\ \quad -2 \quad 8 \quad 8 \\ \hline 1 \quad -4 \quad -4 \quad 0 \end{array}$$

$$x^2 - 4x - 4$$

$$f(x) = x^3 - 8x^2 + 15x + 4; 4$$

$$\frac{4 \pm \sqrt{16 - 4(1)(-4)}}{2(1)} = \frac{4 \pm \sqrt{32}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$$

$$f(x) = x^4 + x^3 - 5x^2 - 4x + 4; 2$$

$$\begin{array}{r} 2 \downarrow \\ 1 \quad 1 \quad -5 \quad -4 \quad 4 \\ \quad 2 \quad 6 \quad 2 \quad -4 \\ \hline 1 \quad 3 \quad 1 \quad -2 \quad 0 \end{array}$$

$$\frac{4 \pm \sqrt{16 - 4(1)(-4)}}{2(1)} = \frac{4 \pm \sqrt{20}}{2}$$

$$f(x) = 2x^3 - 3x^2 - 5x - \frac{3}{2}; -\frac{1}{2}$$

$$\begin{array}{r} -\frac{1}{2} \downarrow \\ 2 \quad -3 \quad -5 \quad -\frac{3}{2} \\ \quad -1 \quad 2 \quad \frac{3}{2} \\ \hline 2 \quad -4 \quad -3 \quad 0 \end{array}$$

$$\frac{4 \pm \sqrt{16 - 4(2)(-3)}}{2(2)} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$$

You try ☺

$$f(x) = x^3 - 4x^2 + 2x + 1; 1$$

$$\begin{array}{r} 1 \downarrow \\ 1 \quad -4 \quad 2 \quad 1 \\ \quad 1 \quad -3 \quad -1 \\ \hline 1 \quad -3 \quad -1 \quad 0 \end{array}$$

$$\frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

$$\begin{array}{r} -2 \downarrow \\ 1 \quad 3 \quad 1 \quad -2 \\ \quad -2 \quad -2 \quad -2 \\ \hline 1 \quad 1 \quad -1 \quad 0 \end{array}$$

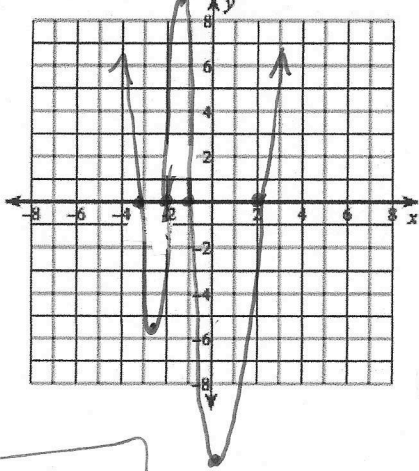
$$\frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

(Finding zeros on next page) ; test points

Identify the zeros, factors, and x-intercepts of each function. Then graph:

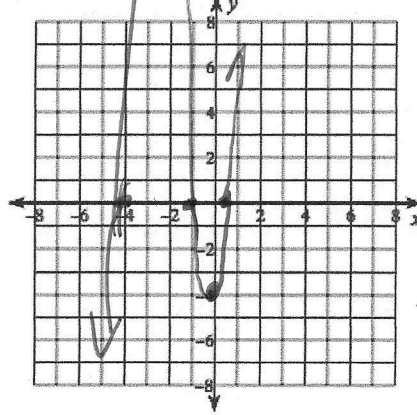
$$f(x) = x^4 + 4x^3 - x^2 - 16x - 12$$



Zeros
-3, -2, -1, 2
End Behavior
↑ ↑
y-int (0, -12)
test points

x	y
-1.5	5.8
-2.5	10.8

$$f(x) = 2x^3 + 9x^2 + 3x - 4$$

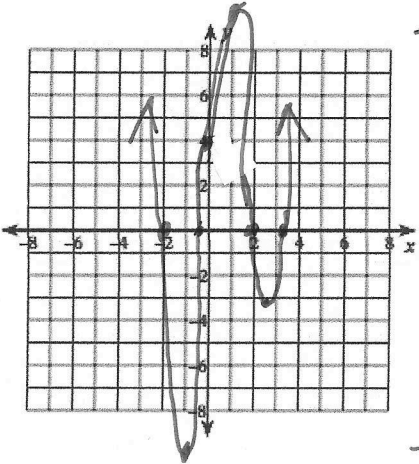


Zeros
-4, -1, 1/2
End Behavior
↓ ↑
y-int (0, -4)
Test points

x	y
-3	14

You try ☺

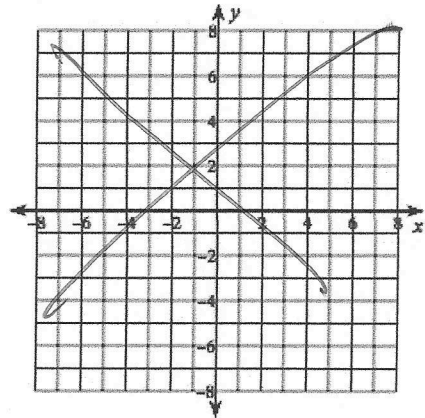
$$f(x) = x^4 - 3x^3 - 5x^2 + 12x + 4$$



Zeros
2, -2, 3.3, -0.3
End Behavior
↑ ↑
y-int (0, 4)
test points

x	y
2.5	-5.1
-1	-9

$$f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$$



I can find a polynomial when given the zeros, intercepts, factors, or roots.

$$c \quad (c, 0) \quad (x - c)$$

Write a polynomial of least degree with integral coefficients that has the given zeros.

-1, 2, 3

$$(x+1)(x-2)(x-3) = x^3 - 4x^2 + x + 6$$

$$(x^2 - x - 2)(x-3) = x^3 - 3x^2 - x^2 + 3x - 2x + 6 = x^3 - 4x^2 + x + 6$$

-2, 2, 4

$$(x+2)(x-2)(x-4) = (x^2 - 4)(x-4) = x^3 - 4x^2 - 4x + 16$$

-3, 1, $\sqrt{2}$, $-\sqrt{2}$

$$(x+3)(x-1)(x-\sqrt{2})(x+\sqrt{2}) = (x^2 + 2x - 3)(x^2 - 2) = x^4 - 2x^2 + 2x^3 - 4x - 3x^2 + 6 = x^4 + 2x^3 - 5x^2 - 4x + 6$$

You try ☺

-3, 1, 2

$$(x+3)(x+1)(x-2) = x^3 - 3x^2 + 2x + 6$$

$$(x+3)(x^2 - 3x + 2) = x^3 - 3x^2 + 9x + 6$$

$$\frac{x^3 - 3x^2 + 2x + 6}{3x^2 - 9x + 6} = x^3 - 7x + 6$$

4, $\sqrt{5}$, $-\sqrt{5}$

$$(x-4)(x-\sqrt{5})(x+\sqrt{5}) = (x-4)(x^2 - 5) = x^3 - 4x^2 - 5x + 20$$

$$x^4 + 2x^3 - 5x^2 - 4x + 6$$

Work to find the zero's on other page

Identify the zeros, factors, and x-intercepts of each function. Then graph:

$$f(x) = x^4 + 4x^3 - x^2 - 16x - 12$$

$$\begin{array}{r} -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} 4 \\ -1 \\ -3 \\ 4 \\ -1 \end{array} \begin{array}{r} -1 \\ -3 \\ 4 \\ 12 \\ 12 \end{array} \begin{array}{r} -12 \\ 12 \\ 0 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} 3 \\ 10 \\ 6 \\ 12 \\ 12 \end{array} \begin{array}{r} -4 \\ -4 \\ 12 \\ 12 \\ 0 \end{array} \begin{array}{r} -12 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

Zeros
-1, 2, -3, 2

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

test point use remainder theorem

$$\begin{array}{r} -1.5 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} 4 \\ -1.5 \\ -3.75 \\ 2.5 \\ -2.75 \end{array} \begin{array}{r} 1 \\ -1.5 \\ -3.75 \\ 4.125 \\ -11.875 \end{array} \begin{array}{r} -12 \\ -12 \\ -12 \\ 17.8125 \\ 5.8 \end{array} \begin{array}{r} -12 \\ 12 \\ 0 \\ 0 \\ 0 \end{array}$$

You try ☺

$$f(x) = x^4 - 3x^3 - 5x^2 + 12x + 4$$

test point
1 | 9

$$\begin{array}{r} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} -3 \\ 1 \\ -2 \\ -7 \\ 5 \end{array} \begin{array}{r} -5 \\ -2 \\ -7 \\ 5 \\ 9 \end{array} \begin{array}{r} 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} \begin{array}{r} 4 \\ 5 \\ 9 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} -3 \\ 2 \\ -2 \\ -14 \\ -4 \end{array} \begin{array}{r} -5 \\ -2 \\ -14 \\ -4 \\ 0 \end{array} \begin{array}{r} 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} \begin{array}{r} 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{array}$$

$$\begin{array}{r} -2 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} -1 \\ -2 \\ 6 \\ -3 \\ -1 \end{array} \begin{array}{r} -7 \\ 6 \\ 2 \\ -1 \\ 0 \end{array} \begin{array}{r} -2 \\ 2 \\ 2 \\ 0 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(-1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

$$\frac{3 - \sqrt{13}}{2} = -0.3$$

$$\frac{3 + \sqrt{13}}{2} = 3.3$$

$$f(x) = 2x^3 + 9x^2 + 3x - 4$$

$$\begin{array}{r} -1 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \begin{array}{r} 9 \\ -2 \\ -7 \\ 7 \\ -4 \end{array} \begin{array}{r} 3 \\ 3 \\ -4 \\ -4 \\ 0 \end{array} \begin{array}{r} -4 \\ 4 \\ 4 \\ 0 \\ 0 \end{array}$$

$$2x^2 + 7x - 4 = 0$$

$$(2x-1)(x+4) = 0$$

Zeros: -1, 1/2, -4

y-int (0, -4)
test points

$$\begin{array}{r} -3 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \begin{array}{r} 9 \\ -6 \\ -9 \\ 3 \\ -6 \end{array} \begin{array}{r} 3 \\ 3 \\ 3 \\ 3 \\ 3 \end{array} \begin{array}{r} -4 \\ 18 \\ 18 \\ 14 \\ 14 \end{array} \begin{array}{r} -4 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

test points

$$\begin{array}{r} -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} -3 \\ -1 \\ -4 \\ -1 \\ -9 \end{array} \begin{array}{r} -5 \\ 4 \\ -1 \\ 13 \\ -9 \end{array} \begin{array}{r} 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} \begin{array}{r} 4 \\ -13 \\ -9 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{r} 2.5 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \begin{array}{r} -3 \\ 2.5 \\ -0.5 \\ -6.25 \\ -3.625 \end{array} \begin{array}{r} -5 \\ -1.25 \\ -6.25 \\ -15.625 \\ -9.0625 \end{array} \begin{array}{r} 12 \\ 12 \\ 12 \\ 12 \\ 12 \end{array} \begin{array}{r} 4 \\ 4 \\ 4 \\ 4 \\ 4 \end{array}$$