

Notes Section 4.3

Find the rational zeros of a polynomial

Using the rational zeros theorem

Using synthetic division

Using the quadratic formula

**THE RATIONAL ZERO THEOREM**

If  $f(x) = a_n x^n + \dots + a_1 x + a_0$  has \_\_\_\_\_ coefficients, then every rational zero of  $f$  has the following form:

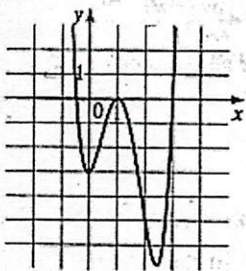
$$\frac{p}{q} = \frac{\text{factor of constant term } \boxed{\phantom{000}}}{\text{factor of leading coefficient } \boxed{\phantom{000}}}$$

1-6 List all possible rational zeros given by the Rational Zeros Theorem  
(But don't check to see which actually are zeros).

2-  $Q(x) = x^4 - 3x^3 - 6x + 8$

4-  $P(x) = 6x^4 - x^2 + 2x + 12$

9-  $P(x) = 2x^4 - 9x^3 + 9x^2 + x - 3$



11-38 Find all the rational zeros of the polynomial:

12-  $P(x) = x^3 - 7x^2 + 14x - 8$

22-  $P(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

34-  $P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2$

You try ☺

23-  $P(x) = x^4 + 6x^3 + 7x^2 - 6x - 8$

39-48 Find all the zeros of the polynomial.  
Use the quadratic formula if necessary.

40.  $P(x) = x^3 - 5x^2 + 2x + 12$

46.  $P(x) = 3x^3 - 5x^2 - 8x - 2$

**DESCARTES' RULE OF SIGNS**

Let  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$   
be a polynomial function with real coefficients.

- The number of \_\_\_\_\_ real zeros of  $f$  is equal to the number of changes in sign of the coefficients of \_\_\_\_\_ or is less than this by an \_\_\_\_\_ number.
- The number of \_\_\_\_\_ real zeros of  $f$  is equal to the number of changes in sign of the coefficients of \_\_\_\_\_ or is less than this by an \_\_\_\_\_ number.

57-62 Use Descartes' Rule of Signs to determine how many positive and negative real zeros the polynomial can have. Then determine the total number of zeros.

58.  $P(x) = 2x^3 - x^2 + 4x - 7$

60.  $P(x) = x^4 + x^3 + x^2 + x + 12$

The Upper and Lower Bounds Theorem:  $a \leq c \leq b$

1.

2.

63-66 Show that the given values for  $a$  and  $b$  are lower and upper bounds for the real zeros of the polynomial.

64.  $P(x) = 2x^3 + 5x^2 + x - 2; a = -3, b = 5$

66.  $P(x) = 3x^4 - 17x^3 + 24x^2 - 9x + 1; a = 0, b = 6$

67-70 Find integers that are upper and lower bounds for the real zeros of the polynomial.

68.  $P(x) = 2x^3 - 3x^2 - 8x + 12$

71-76 Find all rational zeros of the polynomial, and then find the irrational zeros, if any. Whenever appropriate, use the Rational Zeros Theorem, the Upper and Lower Bounds Theorem, Descartes' Rule of Signs, the quadratic formula, or other factoring techniques.

72.  $P(x) = 2x^4 + 15x^3 + 31x^2 + 20x + 4$