

Notes 4.2

Know when to use the Law of Cosines  
 Solve triangles using the Law of Cosines  
 Applications of the Law of Cosines  
 Find the area of a triangle without the height.

Know when to use the Law of Cosines

**LAW OF COSINES**

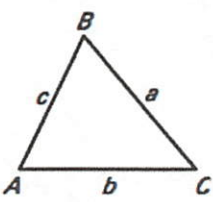
If  $\triangle ABC$  has sides of length  $a$ ,  $b$ , and  $c$  as shown, then:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

(Don't forget to square root)



Solve triangles using the Law of Cosines

Solving a SAS triangle:

- Step 1- Use Law of Cosines to find the side opposite given angle.
- Step 2- Use Law of Sines to find angle opposite shorter of two sides
- Step 3- Find third angle by subtracting from  $180^\circ$

In Exercises 1-14, find the third side of the triangle. Solve the triangle

6)  $a = 122, c = 144, B = 48^\circ$



$$b^2 = 122^2 + 144^2 - 2(122)(144) \cos 48^\circ$$

$$b \approx 110^\circ$$

$$\frac{\sin 48^\circ}{110} = \frac{\sin A}{122}$$

$$\left. \begin{array}{l} \angle A \approx 55.5^\circ \\ \angle C \approx 76.5^\circ \end{array} \right\}$$

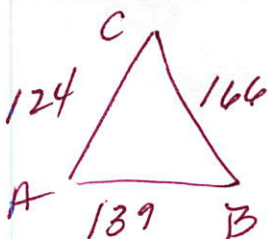
9)  $a = 4.6, b = 7.2, C = 124^\circ$

Solving a SSS triangle:

- Step 1- use Law of Cosines to find angle opposite longest side
- Step 2- use Law of Sines to find either angle
- Step 3- Subtract angles from  $180^\circ$

In Exercises 15-24, given three sides of a triangle, find the specified angle. Solve the triangle

20)  $a = 166, b = 124, c = 139$



$$166^2 = 124^2 + 139^2 - 2(124)(139) \cos A$$

$$-7141 = -34472 \cos A$$

$$.20715 = \cos A$$

$$\frac{\sin 78^\circ}{166} = \frac{\sin B}{124}$$

$$\left. \begin{array}{l} \angle A = 78^\circ \\ \angle B = 47^\circ \\ \angle C = 55^\circ \end{array} \right\}$$

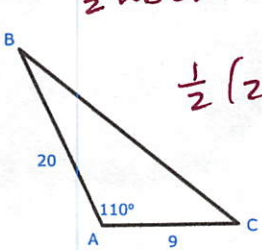
17)  $a = 60, b = 88, c = 120$

**Area of an Oblique Triangle**

**SAS Area**

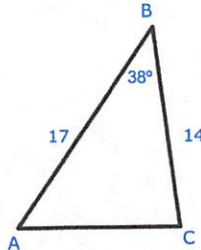
Equals one-half the product of the lengths of two sides times the sine of their included angle.

$K = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B = \frac{1}{2} bc \sin A$



$\frac{1}{2} (20)(9) \sin 110^\circ$

$84.6 \text{ u}^2$



SSS: Area

**HERON'S AREA FORMULA**

The area of a triangle with sides of length  $a, b,$  and  $c$  is

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

where  $s = \frac{a+b+c}{2}$ . The variable  $s$  is called the **semiperimeter**, or half-perimeter, of the triangle.

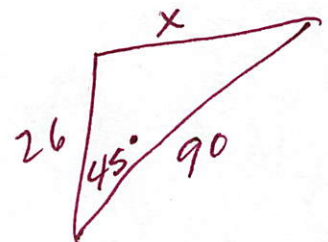
30.  $a = 32, b = 24, c = 36$

$s = \frac{32+24+36}{2} = 46$

$A = \sqrt{46(14)(22)(10)} \approx 376.4 \text{ u}^2$

35.  $a = 3.6, b = 4.2, c = 4.8$

39. **BASEBALL** In a baseball game, a batter hits a ground ball 26 feet in the direction of the pitcher's mound. See the figure at the top of the next column. The pitcher runs forward and reaches for the ball. At that moment, how far is the ball from first base? (Note: A baseball infield is a square that measures 90 feet on each side.)



Picture in book is not a good picture