Notes Section 3.7

Definition of a one-to-one function
- Determine whether a function is one-to-one

Definition of an inverse function:
- Determine whether functions are inverses
- Find the inverse of a function
- Graph the inverse of a function

Definition of a one-to-one function

A function for which every ______________ of the ______________ of the function corresponds to

______________________________ One-to-one is often written 1-1.

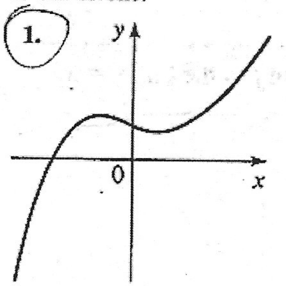
Note: A relation is a function if it passes the ______________________. It is a 1-1 function if it

passes both the ______________________ and the ______________________.

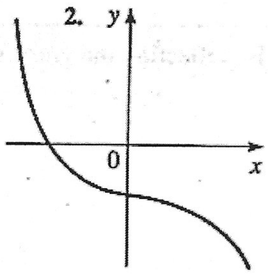
A function, $f(x)$, has an ______________________ if $f(x)$ is one-to-one.

1–6 ■ The graph of a function f is given. Determine whether f is one-to-one.

1.



2.



7-16 Determine whether the function is one-to-one.

8. $f(x) = -2x + 5$

10. $g(x) = |x|$

12. $h(x) = x^3 + 8$

Definition of an inverse function:

A ______________ and its ______________ can be described as the "DO" and the "UNDO" functions.

The "DO" and "UNDO" process can be stated as a composition of functions.
If functions f and g are inverse functions.

Let f be a one-to-one function with ________________________
Then its ***inverse function*** f^{-1} has a ________________________
and is defined by:

____

Don't mistake the -1 in f^{-1} for an exponent:

$$f^{-1}\ does\ not\ mean\ \frac{1}{f(x)}$$

Inverse functions are mirror images over the line ____________

Every ____________ has a ____________ partner.

17-20 **Assume f is a one-to-one function.**

17. (a) $If\ \ f(2) = 7,\ find\ f^{-1}(7).$
(b) $If\ \ f^{-1}(3) = -1,\ find\ f(-1).$

18. (a) $If\ \ f(5) = 18,\ find\ f^{-1}(18).$
(b) $If\ \ f^{-1}(4) = 2,\ find\ f(2).$

21- 30 Use the property of inverse functions to show that f and g *are inverses of each other.*

21. $f(x) = x + 3; \quad g(x) = x - 3$

28. $f(x) = x^3 + 1; \quad g(x) = (x-1)^{\frac{1}{3}}$

23. $f(x) = 2x - 5; \quad g(x) = \frac{x+5}{2}$

How to find the inverse of a function:

Step 1:

Step 2:

Step 3:

Step 4:

________________________**!**

Find the inverse of $f(x) = 2x - 1$

$f(x) = -\frac{2}{3}x + 5$

31-50 Find the inverse function of f.

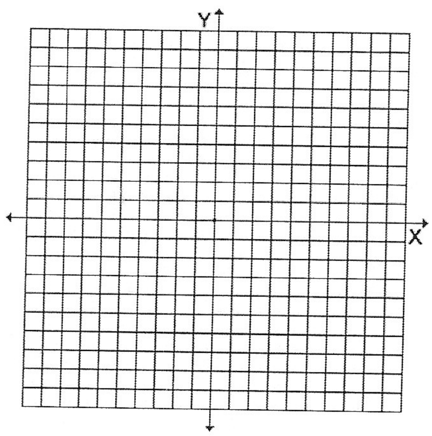
38. $f(x) = \frac{x-2}{x+2}$

44. $f(x) = \sqrt{2x-1}$

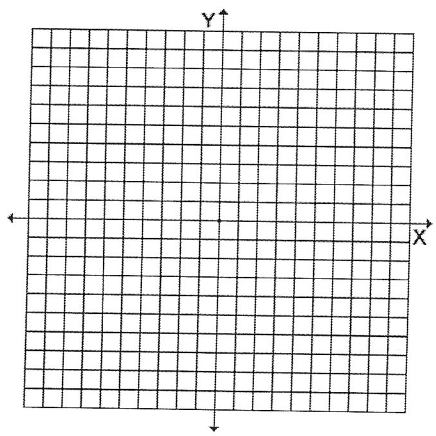
47. $f(x) = 1 + \sqrt{1+x}$

51-54 A function f is given.
(a) Sketch the graph of f
(b) Use the graph of f to sketch the graph of f^{-1}
(c) Find f^{-1}

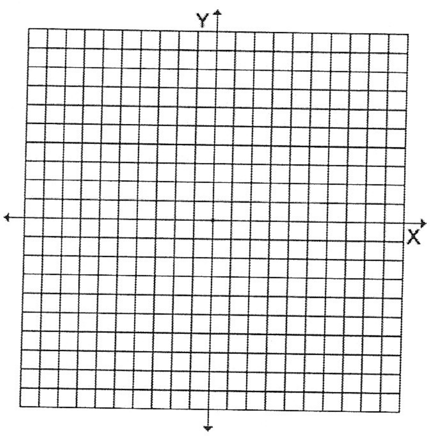
51. $f(x) = 3x - 6$



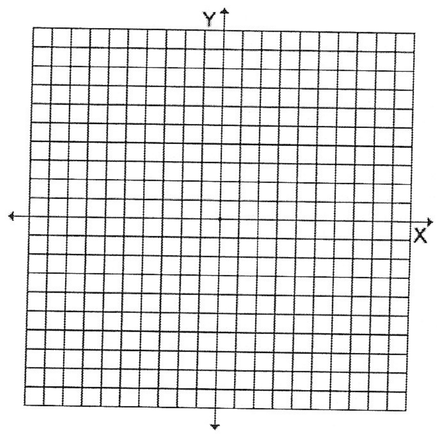
52. $f(x) = 16 - x^2, \quad x \geq 0$



53. $f(x) = \sqrt{x+1}$

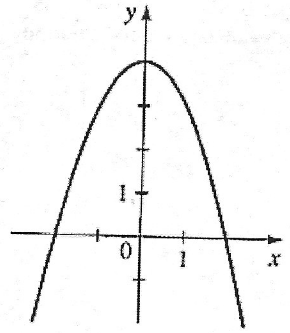


54. $f(x) = x^3 - 1$

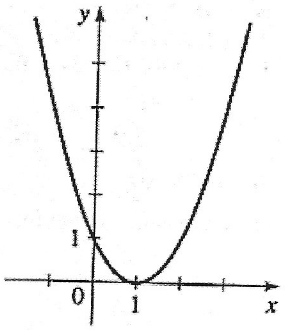


61–64 ▪ The given function is not one-to-one. Restrict its domain so that the resulting function *is* one-to-one. Find the inverse of the function with the restricted domain. (There is more than one correct answer.)

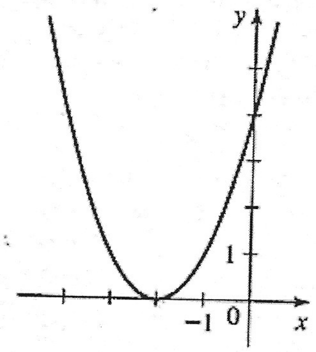
61. $f(x) = 4 - x^2$



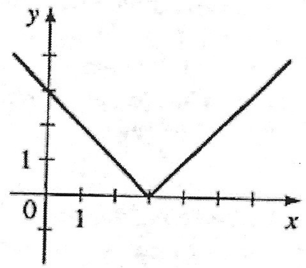
62. $g(x) = (x - 1)^2$



63. $h(x) = (x + 2)^2$

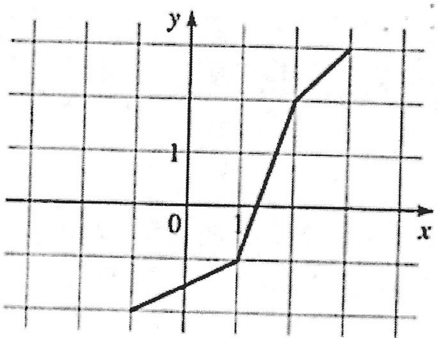


64. $k(x) = |x - 3|$

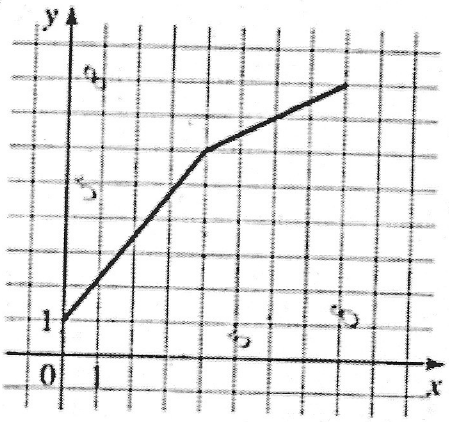


65–66 ▪ Use the graph of f to sketch the graph of f^{-1}.

65.



66.



71. **Temperature Scales** The relationship between the Fahrenheit (F) and Celsius (C) scales is given by

$$F(C) = \tfrac{9}{5}C + 32$$

(a) Find F^{-1}. What does F^{-1} represent?

(b) Find $F^{-1}(86)$. What does your answer represent?

Notes Section 3.7

Definition of a one-to-one function

Determine whether a function is one-to-one

Definition of an inverse function:

Determine whether functions are inverses

Find the inverse of a function

Graph the inverse of a function

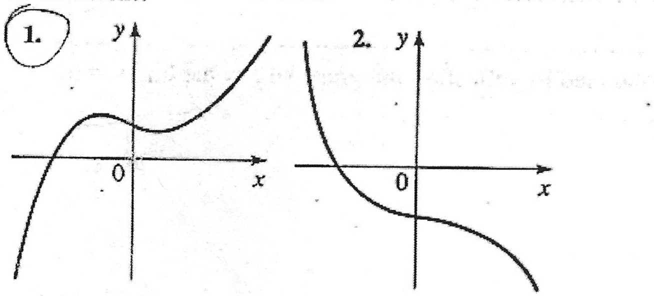
Definition of a one-to-one function

A function for which every element of the range of the function corresponds to exactly one element of the domain One-to-one is often written 1-1.

Note: A relation is a function if it passes the vertical line test. It is a 1-1 function if it passes both the vertical line test and the horizontal line test.

A function, $f(x)$, has an inverse if $f(x)$ is one-to-one.

1–6 ▪ The graph of a function f is given. Determine whether f is one-to-one.



7-16 Determine whether the function is one-to-one.

8. $f(x) = -2x + 5$ yes

10. $g(x) = |x|$ no

12. $h(x) = x^3 + 8$ yes

Definition of an inverse function:

A function and its inverse function can be described as the "DO" and the "UNDO" functions.

The "DO" and "UNDO" process can be stated as a composition of functions.
If functions f and g are inverse functions.

$$f(g(x)) = g(f(x))$$

Let f be a one-to-one function with domain A ; range B
Then its ***inverse function*** f^{-1} has a domain B ; range A
and is defined by:

$f^{-1}(y) = x \longleftrightarrow f(x) = y$

Don't mistake the -1 in f^{-1} for an exponent:

$$f^{-1} \textit{ does not mean } \frac{1}{f(x)}$$

Inverse functions are mirror images over the line $y = x$

Every (x, y) has a (y, x) partner.

17-20 **Assume f is a one-to-one function.**

17. (a) If $f(2) = 7$, find $f^{-1}(7)$. $= 2$
(b) If $f^{-1}(3) = -1$, find $f(-1)$. $= 3$

18. (a) If $f(5) = 18$, find $f^{-1}(18)$. $= 5$
(b) If $f^{-1}(4) = 2$, find $f(2)$. $= 4$

21- 30 Use the property of inverse functions to show that f and g *are inverses of each other.*

21. $f(x) = x + 3; \quad g(x) = x - 3$

$f(g(x)) = f(x-3) = (x-3)+3 = x$
$g(f(x)) = g(x+3) = (x+3) - 3 = x$

28. $f(x) = x^3 + 1; \quad g(x) = (x-1)^{\frac{1}{3}}$

$f(g(x)) = f((x-1)^{1/3}) = ((x-1)^{1/3})^3 + 1 = x - 1 + 1 = x$
$g(f(x)) = g(x^3+1) = (x^3+1-1)^{1/3} = (x^3)^{1/3} = x$

23. $f(x) = 2x - 5; \quad g(x) = \frac{x+5}{2}$

How to find the inverse of a function:

Step 1: Stick y in for f(x)

Step 2: Switch x and y

Step 3: Solve for y

Step 4: Stick $f^{-1}(x)$ in for y

Check it !

Find the inverse of $f(x) = 2x - 1$

$y = 2x - 1$
$x = 2y - 1$
$x + 1 = 2y$
$y = \frac{x+1}{2}$ or $\frac{1}{2}x + \frac{1}{2}$

$f(\frac{1}{2}x + \frac{1}{2})$
$= 2(\frac{1}{2}x + \frac{1}{2}) - 1$
$x + 1 - 1 = \boxed{x}$

$f(x) = -\frac{2}{3}x + 5$

$y = -\frac{2}{3}x + 5$
$x = -\frac{2}{3}y + 5$
$x - 5 = -\frac{2}{3}y$
$-3/2(x-5) = y$

$-\frac{3}{2}x + \frac{15}{2}$

$-\frac{2}{3}(-\frac{3}{2}x + \frac{15}{2}) + 5$
$x - 5 + 5 = x$

31-50 Find the inverse function of f.

38. $f(x) = \frac{x-2}{x+2}$

$y = \frac{x-2}{x+2}$ $\quad x = \frac{y-2}{y+2}$

$x(y+2) = y-2$

$xy + 2x = y - 2$

$xy - y = -2x - 2$

$y(x-1) = -2x-2$

$y = \frac{-2x-2}{x-1}$

$\frac{\frac{-2x-2}{x-1} - 2}{\frac{2x-2}{x-1} + 2}$ $\quad \frac{-2x-2-2x+2}{-2x-2+2x-2}$ $\quad \frac{-4x}{-4} = x$

44. $f(x) = \sqrt{2x-1}$

$x = \sqrt{2y-1}$

$x^2 = 2y-1$

$x^2+1 = 2y$

$y = \frac{x^2+1}{2}$

$f^{-1}(x) = \frac{x^2+1}{2}$

$f(f^{-1}(x)) = \sqrt{2\left(\frac{x^2+1}{2}\right)-1}$

$\sqrt{x^2+1-1} = \sqrt{x^2} = x$

47. $f(x) = 1 + \sqrt{1+x}$

$x = 1 + \sqrt{1+y}$

$x - 1 = \sqrt{1+y}$

$1 + y = (x-1)^2$

$y = (x-1)^2 - 1$

$f(f^{-1}(x)) = 1 + \sqrt{1+x}$

$1 + \sqrt{1+(x-1)^2-1} = 1 + \sqrt{(x-1)^2} = 1 + x - 1$

$= x$

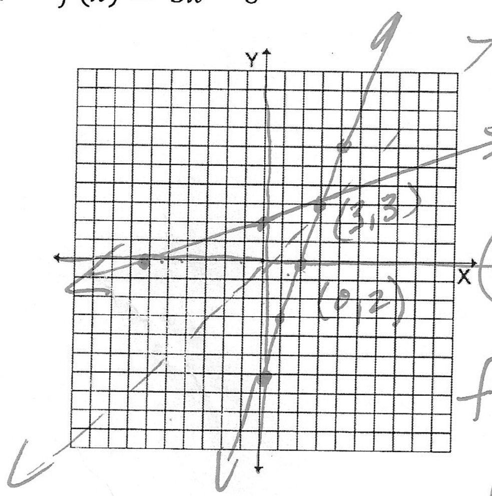
51-54 A function f is given.

(a) Sketch the graph of f

(b) Use the graph of f to sketch the graph of f^{-1}

(c) Find f^{-1}

51. $f(x) = 3x - 6$



$y = 3x - 6$

$x = 3y - 6$

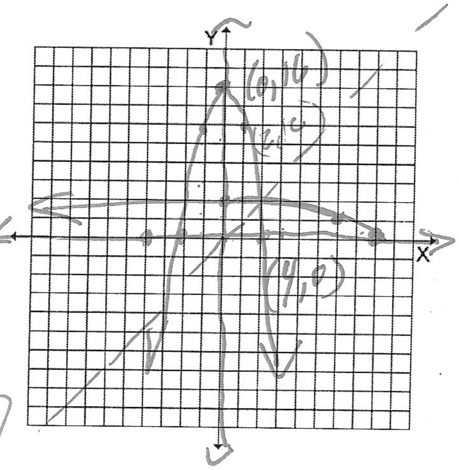
$x + 6 = 3y$

$y = \frac{x+6}{3}$

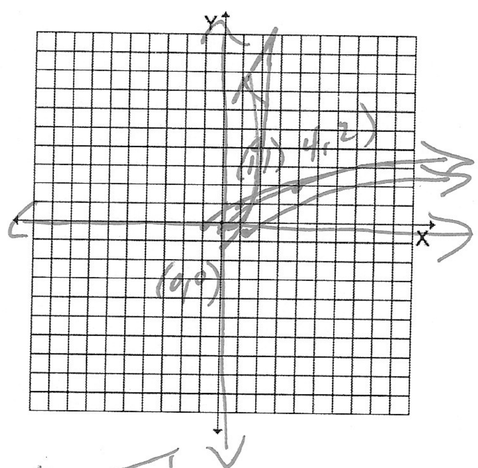
$f^{-1}(x) = \frac{x+6}{3}$

$= \frac{1}{3}x + 2$

52. $f(x) = 16 - x^2, \quad x \geq 0$



53. $f(x) = \sqrt{x+1}$



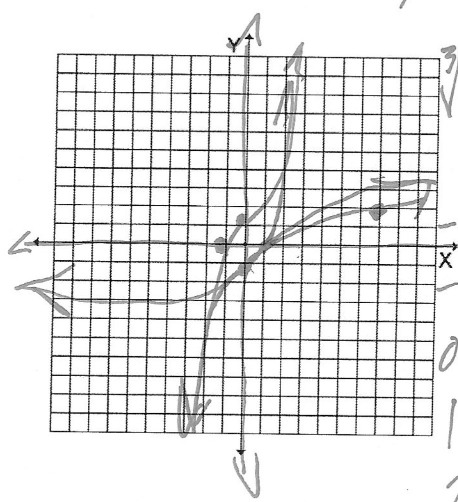
$x = \sqrt{y+1}$

$x^2 = y+1$ $\quad y = x^2 - 1$

54. $f(x) = x^3 - 1$

$x = y^3 - 1$

$\sqrt[3]{x+1} = f^{-1}(x)$



x	y
-2	-9
-1	-2
0	-1
1	0
2	7

x	$f^{-1}(x)$
-9	-2
-2	-1
-1	0
0	1
7	2

61–64 ■ The given function is not one-to-one. Restrict its domain so that the resulting function *is* one-to-one. Find the inverse of the function with the restricted domain. (There is more than one correct answer.)

$\sqrt{x}-2=y$

$x=(y+2)^2$

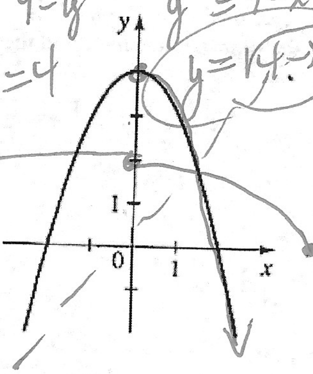
$y=|x-3|$

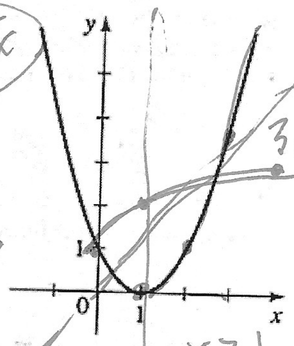
$x=|y-3|$

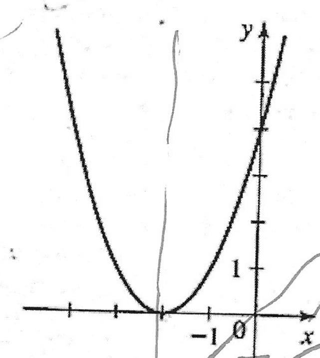
61. $f(x) = 4 - x^2$ **62.** $g(x) = (x-1)^2$ **63.** $h(x) = (x+2)^2$ **64.** $k(x) = |x-3|$

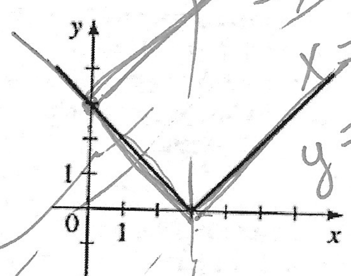
$x=4-y^2$ $y^2=4-x$

$y^2+x=4$ $y=\sqrt{4-x}$









$-x=y-3$

$x=y-3$

$y=x+3$

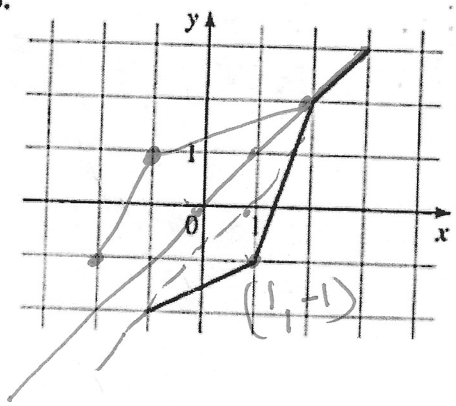
$x\geq 0$ $x\geq 1$ $x\geq -2$ $x\geq 3$

$x=(y-1)^2$

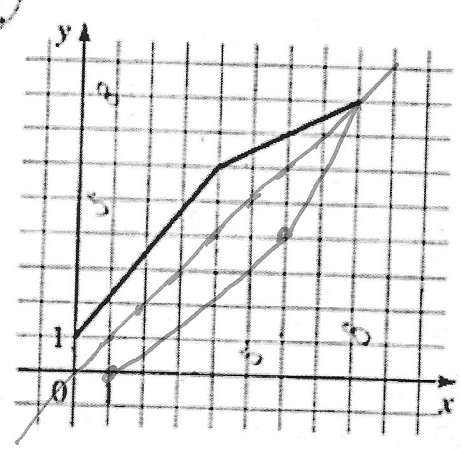
$\sqrt{x}+1=y$

65–66 ■ Use the graph of f to sketch the graph of f^{-1}.

65.



66.



$y=x-3$

$x=y-3$

$y=x+3$

$y=-x+3$

$x=-y+3$

$y+x=+3$

$y=-x+3$

$x-3+3$

$-(-x+3)+3$

$y=-x+3$

$x=-y+3$

$x+y=3$

$\boxed{y=-x+3}$

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$$F(C) = \tfrac{9}{5}C + 32$$

(a) Find F^{-1}. What does F^{-1} represent?

(b) Find $F^{-1}(86)$. What does your answer represent?

$f^{-1}(c)=\frac{5}{9}(c-32)$

Changing f to c

$y=\frac{9}{5}c+32$

$c=\frac{9}{5}y+32$

$c-32=\frac{9}{5}y$

$y=\frac{5}{9}(c-32)$

$\frac{5}{9}(86-32)$

$=30$

$30°$ Celcius