

Notes Section 3.6:

Combining Functions

Find the Domain of the Combining of Two Functions

Composition of Functions

Algebra of Functions:

$$(f + g)(x) =$$

$$(f - g)(x) =$$

$$(fg)(x) =$$

$$\left(\frac{f}{g}\right)(x) =$$

**Combining Functions**

$$f(x) = \frac{1}{x-2} \text{ and } g(x) = \sqrt{x}$$

Find  $(f + g)(4)$ ,  $(f - g)(4)$ ,  $(fg)(4)$ , and  $\left(\frac{f}{g}\right)(4)$

**Find the Domain of the Combining of Two Functions**

Combinations of Functions and their Domains

$$f(x) = \frac{1}{x-2} \text{ and } g(x) = \sqrt{x}$$

Find the functions of

$f + g$ ,  $f - g$ ,  $fg$ , and  $\frac{f}{g}$  and their domains.

1-6 ■ Find  $f + g$ ,  $f - g$ ,  $fg$ , and  $f/g$  and their domains.

$$\textcircled{3} f(x) = \sqrt{1+x^2}, \quad g(x) = \sqrt{1-x}$$

**Composition of Functions**

Given two functions  $f$  and  $g$ , the **composite function**  $(f \circ g)$  is defined by:

$$f(x) = -4x + 3 \text{ and } g(x) = -2x^2 - 6$$

Evaluate the expression

$$(f \circ g)(3) \text{ and } (g \circ f)(3)$$

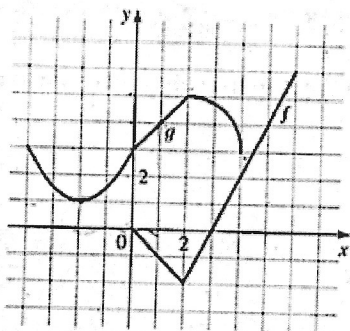
19-

$$f(x) = 3x - 5 \text{ and } g(x) = 2 - x^2$$

Evaluate the expression

$$(f \circ g)(-2) \text{ and } (g \circ f)(-2)$$

23–28 ■ Use the given graphs of  $f$  and  $g$  to evaluate the expression.



23.  $f(g(2))$

24.  $g(f(0))$

25.  $(g \circ f)(4)$

26.  $(f \circ g)(0)$

29–40 ■ Find the functions  $f \circ g$ ,  $g \circ f$ ,  $f \circ f$ , and  $g \circ g$  and their domains.

29.  $f(x) = 2x + 3$ ,  $g(x) = 4x - 1$

33.  $f(x) = \frac{1}{x}$ ,  $g(x) = 2x + 4$

41–44 ■ Find  $f \circ g \circ h$ .

41.  $f(x) = x - 1$ ,  $g(x) = \sqrt{x}$ ,  $h(x) = x - 1$

42.  $f(x) = \frac{1}{x}$ ,  $g(x) = x^3$ ,  $h(x) = x^2 + 2$

45–50 ■ Express the function in the form  $f \circ g$ .

45.  $F(x) = (x - 9)^5$

46.  $F(x) = \sqrt{x} + 1$

47.  $G(x) = \frac{x^2}{x^2 + 4}$

48.  $G(x) = \frac{1}{x + 3}$

Notes Section 3.6:

Combining Functions

Find the Domain of the Combining of Two Functions

Composition of Functions

Algebra of Functions:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Combining Functions

$$f(x) = \frac{1}{x-2} \text{ and } g(x) = \sqrt{x}$$

$$(f+g)(4) = \frac{1}{2} + 2 = 2\frac{1}{2}$$

$$(f-g)(4) = \frac{1}{2} - 2 = -1\frac{1}{2}$$

$$fg(4) = \frac{1}{2}(2) = 1$$

Find  $(f + g)(4)$ ,  $(f - g)(4)$ ,  $(fg)(4)$ , and  $\left(\frac{f}{g}\right)(4)$

$$\frac{f}{g}(4) = \frac{1/2}{2} = \frac{1}{4}$$

Find the Domain of the Combining of Two Functions

Combinations of Functions and their Domains

$$f(x) = \frac{1}{x-2} \text{ and } g(x) = \sqrt{x}$$

$x \neq 2$                        $x \geq 0$

$$f+g = [0, 2) \cup (2, \infty)$$

$$f-g = [0, 2) \cup (2, \infty)$$

$$f \cdot g = [0, 2) \cup (2, \infty)$$

$$\frac{f}{g}(x) = (0, 2) \cup (2, \infty)$$

Find the functions of

$f + g$ ,  $f - g$ ,  $fg$ , and  $\frac{f}{g}$  and their domains.

1-6 ■ Find  $f + g$ ,  $f - g$ ,  $fg$ , and  $f/g$  and their domains.

$$\textcircled{3} f(x) = \sqrt{1+x^2}, \quad g(x) = \sqrt{1-x}$$

$(-\infty, \infty)$      $(-\infty, -1]$      $\begin{matrix} 1-x \geq 0 \\ -x \geq -1 \\ x \leq -1 \end{matrix}$

$$\left. \begin{matrix} f+g \\ f-g \\ fg \end{matrix} \right\} (-\infty, -1] \quad \frac{f}{g} (-\infty, -1)$$

Composition of Functions

Given two functions  $f$  and  $g$ , the **compositie function**  $(f \circ g)$  is defined by:

$$(f \circ g)(x) = f(g(x))$$

$$f(x) = -4x + 3 \text{ and } g(x) = -2x^2 - 6$$

Evaluate the expression

$$(f \circ g)(3) \text{ and } (g \circ f)(3)$$

$$f(g(3)) = f(-24) = \boxed{119}$$

$$g(f(3)) = g(-9) = \boxed{-168}$$

19-

$$f(x) = 3x - 5 \text{ and } g(x) = 2 - x^2$$

Evaluate the expression

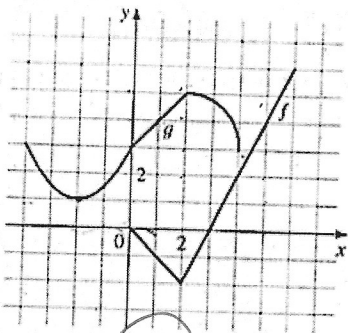
$$(f \circ g)(-2) \text{ and } (g \circ f)(-2)$$

$$f(g(-2)) = f(-2) = \boxed{-11}$$

$$g(f(-2)) = g(-11) = \boxed{-119}$$

$2 - (-11)^2$

23-28 ■ Use the given graphs of  $f$  and  $g$  to evaluate the expression.



23.  $f(g(2))$

$f(2) = 4$

24.  $g(f(0))$

$g(0) = 3$

25.  $(g \circ f)(4)$

$g(f(4)) = g(2) = 4$

26.  $(f \circ g)(0)$

$f(g(0)) = f(3) = 0$

29-40 ■ Find the functions  $f \circ g$ ,  $g \circ f$ ,  $f \circ f$ , and  $g \circ g$  and their domains.

29.  $f(x) = 2x + 3$ ,  $g(x) = 4x - 1$

$f(g(x)) = f(4x-1) = 2(4x-1) + 3 = 8x - 2 + 3 = 8x + 1$   $(-\infty, \infty)$

$g(f(x)) = g(2x+3) = 4(2x+3) - 1 = 8x + 12 - 1 = 8x + 11$   $(-\infty, \infty)$

$f(f(x)) = f(2x+3) = 2(2x+3) + 3 = 4x + 6 + 3 = 4x + 9$   $(-\infty, \infty)$

$g(g(x)) = g(4x-1) = 4(4x-1) - 1 = 16x - 4 - 1 = 16x - 5$   $(-\infty, \infty)$

33.  $f(x) = \frac{1}{x}$ ,  $g(x) = 2x + 4$

$f(g(x)) = f(2x+4) = \frac{1}{2x+4}$   $(-\infty, -2) \cup (-2, \infty)$

$g(f(x)) = g(\frac{1}{x}) = 2(\frac{1}{x}) + 4 = \frac{2}{x} + 4$   $(-\infty, 0) \cup (0, \infty)$

$f(f(x)) = f(\frac{1}{x}) = \frac{1}{\frac{1}{x}} = x$   $(-\infty, 0) \cup (0, \infty)$

$g(g(x)) = g(2x+4) = 2(2x+4) + 4 = 4x + 8 + 4 = 4x + 12$   $(-\infty, \infty)$

41-44 ■ Find  $f \circ g \circ h$ .

41.  $f(x) = x - 1$ ,  $g(x) = \sqrt{x}$ ,  $h(x) = x - 1$

$f(g(h(x))) = f(g(x-1)) = f(\sqrt{x-1}) = \sqrt{x-1} - 1$

45)  $f(x) = x^5$   
 $g(x) = x - 9$   
 $f(g(x))$

45-50 ■ Express the function in the form  $f \circ g$ .

45.  $F(x) = (x - 9)^5$

46.  $F(x) = \sqrt{x} + 1$

47.  $G(x) = \frac{x^2}{x^2 + 4}$

48.  $G(x) = \frac{1}{x+3}$

46)  $f(x) = x + 1$   
 $g(x) = \sqrt{x}$

48)  $f(x) = \frac{1}{x}$   
 $g(x) = x + 3$

47)  $f(x) = \frac{x}{x+4}$   
 $g(x) = x^2$