

## Notes 3.1B Solving Systems of Equations

- **Solve systems of equations by substitution**
- **Solve systems of equations by elimination**

The Substitution Method:

Step 1:

Step 2:

Step 3:

Step 4:

**Example 1 Use the substitution method**

Solve the system using the substitution method

$$x + 2y = -2$$

$$3x + 4y = 6$$

*Use the substitution method to solve the system of equations:*

1-  $x^2 - y = 1$   
 $2x^2 + 3y = 17$

2-  $3x + y = 8$   
 $y = x^2 - 5x$

3-  $x^2 + y = 2$   
 $x^2 - 3x - y = 0$

4-  $x - y^2 = 4$   
 $x - y = 2$

The Elimination Method:

Step 1:

Step 2:

Step 3:

Step 4:

**Example 2 Use the elimination method**

Solve the system using the substitution method

$$2x + 5y = 14$$

$$4x + 2y = -4$$

Use the elimination method to solve the system of equations:

$$\begin{aligned} 1- \quad x^2 - y^2 &= 1 \\ 2x^2 - y^2 &= x + 3 \end{aligned}$$

$$\begin{aligned} 2- \quad x^2 + y^2 &= 4x \\ x &= y^2 \end{aligned}$$

$$\begin{aligned} 3- \quad x^2 + y^2 &= 10 \\ x^2 + 2y^2 - 7y &= 0 \end{aligned}$$

$$\begin{aligned} 4- \quad x + y^2 &= 4 \\ x - y &= 2 \end{aligned}$$

### Example 3 Solve linear systems with many or no solutions

$$\begin{aligned} x - 3y &= 7 \\ 2x - 6y &= 12 \end{aligned}$$

$$\begin{aligned} 2x - 6y &= 12 \\ -5x + 15y &= -30 \end{aligned}$$

### Modeling linear equations:

Find the value of two numbers if their sum is 12 and their difference is 4.

The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Brenda's school is selling tickets to a spring musical. On the first day of ticket sales the school sold 3 senior citizen tickets and 9 child tickets for a total of \$75. The school took in \$67 on the second day by selling 8 senior citizen tickets and 5 child tickets. What is the price each of one senior citizen ticket and one child ticket?

## Notes 3.1B Solving Systems of Equations

- Solve systems of equations by substitution
- Solve systems of equations by elimination

The Substitution Method:

- Step 1: Solve the equation for one of its variables
- Step 2: Substitute into the other equation
- Step 3: Solve for the variables
- Step 4: Check your answer

### Example 1 Use the substitution method

Solve the system using the substitution method

$$x + 2y = -2 \rightarrow x = -2y - 2$$

$$3x + 4y = 6$$

$$\begin{aligned} \hookrightarrow 3(-2y - 2) + 4y &= 6 \\ -6y - 6 + 4y &= 6 \\ -2y &= 12 \end{aligned}$$

$$\begin{aligned} x &= -2(-6) - 2 \\ &= -10 \end{aligned}$$

$$\boxed{(-10, -6)}$$

Use the substitution method to solve the system of equations:

1-  $x^2 - y = 1$

$$2x^2 + 3y = 17$$

$$2x^2 + 3(x^2 - 1) = 17$$

$$2x^2 + 3x^2 - 3 = 17$$

$$5x^2 = 20$$

$$y = x^2 - 1$$

2-  $y = 4 - 1$

$$= 3$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\boxed{(2, 3), (-2, 3)}$$

3-  $x^2 + y = 2$

$$x^2 - 3x - y = 0$$

2-  $3x + y = 8$

$$y = x^2 - 5x$$

$$3x + (x^2 - 5x) = 8$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$\boxed{x = -2, 4}$$

4-  $x - y^2 = 0$

$$x - y = 2$$

$$x = y^2$$

$$\boxed{(4, 2) (1, -1)}$$

$$y^2 - y = 2$$

$$y^2 - y - 2 = 0$$

$$(y - 2)(y + 1) = 0$$

$$y = 2 \quad y = -1$$

The Elimination Method:

- Step 1: multiply one or both of the equations to get coefficients that differ in sign for one of the variables.
- Step 2: Add the revised equations. Combining like terms eliminates one of the variables. Solve for other variable.
- Step 3: Substitute the solved variable into one of the original equations and solve for the other variable.
- Step 4: Check your solution

### Example 2 Use the elimination method

Solve the system using the substitution method

$$(2x + 5y = 14) \cdot 2$$

$$4x + 2y = -4$$

$$-4x - 10y = -28$$

$$4x + 2y = -4$$

$$\hline -8y = -32$$

$$y = 4$$

$$2x + 5(4) = 14$$

$$2x = 10$$

$$x = 5$$

$$\boxed{(5, 4)}$$

Use the elimination method to solve the system of equations:

1- 
$$\begin{aligned} x^2 - y^2 &= 1 \\ 2x^2 - y^2 &= x + 3 \\ \hline -x^2 + y^2 &= -1 \\ \hline x^2 &= x + 2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 1 \text{ or } 2 \end{aligned}$$

$$\begin{aligned} x^2 - y^2 &= 1 \\ -y^2 &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x^2 - y^2 &= 1 \\ -y^2 &= -1 \\ y &= \pm 1 \end{aligned}$$

Solutions:  $(1, 0)$ ,  $(2, \pm 1)$

2- 
$$\begin{aligned} x^2 + y^2 &= 4x \\ x &= y^2 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 &= 4x \\ x - y^2 &= 0 \\ \hline 2x^2 &= 4x \\ 2x^2 - 4x &= 0 \\ 2x(x-2) &= 0 \\ x &= 0 \text{ or } x=2 \end{aligned}$$

Solutions:  $(0, 0)$ ,  $(2, \pm\sqrt{2})$

3- 
$$\begin{aligned} x^2 + y^2 &= 10 \\ x^2 + 2y^2 - 7y &= 0 \end{aligned}$$

4- 
$$\begin{aligned} x + y^2 &= 4 \\ x - y &= 2 \end{aligned}$$

### Example 3 Solve linear systems with many or no solutions

$$\begin{aligned} x - 3y &= 7 & x &= 3y + 7 \\ 2x - 6y &= 12 \end{aligned}$$

$$\begin{aligned} 2(3y+7) - 6y &= 12 \\ 6y + 14 - 6y &= 12 \\ 14 &= 12 \text{ False} \end{aligned}$$

$$\begin{aligned} (2x - 6y = 12) / 2 &\Rightarrow x - 3y = 6 \\ (-5x + 15y = -30) / 5 &\Rightarrow -x + 3y = -6 \end{aligned}$$

$$\begin{aligned} x - 3y &= 6 \\ -x + 3y &= -6 \\ \hline 0 &= 0 \text{ True} \end{aligned}$$

no solution

infinite solutions

### Modeling linear equations:

Find the value of two numbers if their sum is 12 and their difference is 4.

$$\begin{aligned} x + y &= 12 \\ x - y &= 4 \\ \hline 2x &= 16 \\ x &= 8 \\ y &= 4 \end{aligned}$$

The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Vans (x)      buses (y)

$$\begin{aligned} (8x + 8y = 240) / 8 &\Rightarrow x + y = 30 \\ (4x + y = 54) / (-1) &\Rightarrow -4x - y = -54 \\ \hline -3x &= -24 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} 4(8) + y &= 54 \\ y &= 22 \end{aligned}$$

8<sup>th</sup> vans  
22<sup>nd</sup> buses

Brenda's school is selling tickets to a spring musical. On the first day of ticket sales the school sold 3 senior citizen tickets and 9 child tickets for a total of \$75. The school took in \$67 on the second day by selling 8 senior citizen tickets and 5 child tickets. What is the price each of one senior citizen ticket and one child ticket?

Senior (x)      child (y)

$$\begin{aligned} (3x + 9y = 75) / 3 &\Rightarrow x + 3y = 25 \\ (8x + 5y = 67) / (-3) &\Rightarrow -8x - 15y = -201 \\ \hline 57y &= 399 \\ y &= 7 \end{aligned}$$

$$\begin{aligned} 3x + 9(7) &= 75 \\ 3x + 63 &= 75 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

\$4 Senior  
\$7 Child