

11.9 Inverse Trigonometric Functions

I can find the value of an angle by using inverse trig functions on my calculator and using the unit circle.
I understand the restrictions on the domain for inverse functions.

Vocabulary:

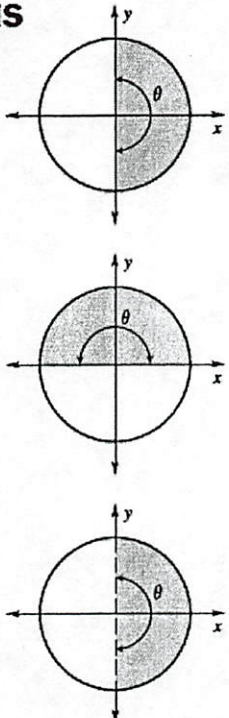
Cosine Inverse:

Sine Inverse:

Principal Values:

INVERSE TRIGONOMETRIC FUNCTIONS

- If $-1 \leq a \leq 1$, then the _____ of a is an angle θ , written $\theta = \sin^{-1} a$, where $\sin \theta = a$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ (or $-90^\circ \leq \theta \leq 90^\circ$).
- If $-1 \leq a \leq 1$, then the _____ of a is an angle θ , written $\theta = \cos^{-1} a$, where $\cos \theta = a$ and $0 \leq \theta \leq \pi$ (or $0^\circ \leq \theta \leq 180^\circ$).
- If a is any real number, then the _____ of a is an angle θ , written $\theta = \tan^{-1} a$, where $\tan \theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90^\circ < \theta < 90^\circ$).



Checkpoint Evaluate the expression in both radians and degrees.

Example 1 Evaluate inverse trigonometric functions

Evaluate the expression in both radians and degrees.

- a. $\cos^{-1} 3$ b. $\tan^{-1} \frac{\sqrt{3}}{3}$

Solution

a. There is _____ whose cosine is 3. So, $\cos^{-1} 3$ is _____

b. When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90^\circ < \theta < 90^\circ$, the angle whose tangent is $\frac{\sqrt{3}}{3}$ is:

$\theta = \tan^{-1} \frac{\sqrt{3}}{3} = \underline{\hspace{1cm}}$ or $\theta = \tan^{-1} \frac{\sqrt{3}}{3} = \underline{\hspace{1cm}}$

1. $\sin^{-1} \frac{\sqrt{3}}{2}$

2. $\cos^{-1}(-0.5)$

3. $\tan^{-1}(-1)$

You try ☺

Evaluate each trigonometric function. Answer in radians and degrees

1) $\cos^{-1} \frac{\sqrt{2}}{2}$

2) $\tan^{-1} \frac{\sqrt{3}}{3}$

3) $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right)$

4) $\cos^{-1} 0$

5) $\sec^{-1} 2$

6) $\csc^{-1} 1$

7) $\cot^{-1} -1$

8) $\sin^{-1} \frac{\sqrt{3}}{2}$

9) $\tan^{-1} \left(-\frac{\sqrt{3}}{3} \right)$

10) $\cos^{-1} -\frac{1}{2}$

Using the calculator ☺

Find each angle measure to the nearest degree.

1) $\sin B = 0.4848$

2) $\sin A = 0.5150$

Review:

Find the exact value of each trigonometric function.

1) $\sin 300^\circ$

2) $\cos \frac{\pi}{3}$

3) $\tan 240^\circ$

4) $\sin -\frac{5\pi}{6}$

11.9 Inverse Trigonometric Functions

I can find the value of an angle by using inverse trig functions on my calculator and using the unit circle.
I understand the restrictions on the domain for inverse functions.

Vocabulary:

Cosine Inverse: *The inverse of $y = \cos x$, $\cos^{-1} y = x$*

Sine Inverse: *The inverse of $y = \sin x$, $\sin^{-1} y = x$*

Principal Values:

The values in the restricted domains of trigonometric functions.

INVERSE TRIGONOMETRIC FUNCTIONS

- If $-1 \leq a \leq 1$, then the inverse sine of a is an angle θ , written $\theta = \sin^{-1} a$, where $\sin \theta = a$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ (or $-90^\circ \leq \theta \leq 90^\circ$).
- If $-1 \leq a \leq 1$, then the inverse cosine of a is an angle θ , written $\theta = \cos^{-1} a$, where $\cos \theta = a$ and $0 \leq \theta \leq \pi$ (or $0^\circ \leq \theta \leq 180^\circ$).
- If a is any real number, then the inverse tangent of a is an angle θ , written $\theta = \tan^{-1} a$, where $\tan \theta = a$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (or $-90^\circ < \theta < 90^\circ$).

✔ **Checkpoint** Evaluate the expression in both radians and degrees.

Example 1 Evaluate inverse trigonometric functions

Evaluate the expression in both radians and degrees.

a. $\cos^{-1} 3$ b. $\tan^{-1} \frac{\sqrt{3}}{3}$

Solution

a. There is no angle whose cosine is 3. So, $\cos^{-1} 3$ is undefined.

b. When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90^\circ < \theta < 90^\circ$, the angle whose tangent is $\frac{\sqrt{3}}{3}$ is:

$\theta = \tan^{-1} \frac{\sqrt{3}}{3} = \frac{\pi}{6}$ or $\theta = \tan^{-1} \frac{\sqrt{3}}{3} = 30^\circ$

1. $\sin^{-1} \frac{\sqrt{3}}{2}$
 $\frac{\pi}{3}, 60^\circ$

2. $\cos^{-1}(-0.5)$
 $\frac{2\pi}{3}, 120^\circ$

3. $\tan^{-1}(-1)$
 $-\frac{\pi}{4}, -45^\circ$

You try ☺

Evaluate each trigonometric function. Answer in radians and degrees

1) $\cos^{-1} \frac{\sqrt{2}}{2}$ $45^\circ, \pi/4$

2) $\tan^{-1} \frac{\sqrt{3}}{3}$ $30^\circ, \pi/6$

3) $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right)$ $-60^\circ, -\frac{\pi}{3}$
Handwritten note: $\frac{1}{2}, \frac{\sqrt{3}}{2}$ with a right triangle diagram.

4) $\cos^{-1} 0$
 $90^\circ, \pi/2$

5) $\sec^{-1} 2$
 $\cos^{-1} \left(\frac{1}{2} \right)$ $60^\circ, \pi/3$

6) $\csc^{-1} 1$
 $\sin^{-1}(1) = \frac{\pi}{2}$ or 90°

7) $\cot^{-1} -1$
 $-\frac{\pi}{4}, 45^\circ$

8) $\sin^{-1} \frac{\sqrt{3}}{2}$ $\frac{\pi}{3}, 60^\circ$

9) $\tan^{-1} \left(-\frac{\sqrt{3}}{3} \right)$ $-\frac{\pi}{6}, -30^\circ$

10) $\cos^{-1} -\frac{1}{2}$ $\frac{2\pi}{3}, 120^\circ$

Using the calculator ☺

Find each angle measure to the nearest degree.

1) $\sin B = 0.4848$ 29°

2) $\sin A = 0.5150$

31°

Review:

Find the exact value of each trigonometric function.

1) $\sin 300^\circ$
 $-\frac{\sqrt{3}}{2}$

2) $\cos \frac{\pi}{3}$ $\frac{1}{2}$

3) $\tan 240^\circ$ $\sqrt{3}$

4) $\sin -\frac{5\pi}{6}$ $-\frac{1}{2}$

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:

