Notes Section **10.2 Permutations and Combinations**

- I know the difference between a permutation and a combination.
- I can use the permutation rule to find the arrangement of objects in a specific order
- I can use the combination rule to find the arrangement of objects when order doesn't matter

PERMUTATIONS and COMBINATIONS

If the order doesn't matter, it is a **Combination.**
If the order **does** matter it is a **Permutation.**

Permutations:

A permutation is used when re-arranging the elements of the set creates a new situation.

To determine the number of permutations we use the rule:

Permutation:

How many ways could we get 1st, 2nd, and 3rd place winners in a race with the following 4 people? Jay, Sue, Kim, and Bob

Combinations:

Suppose that you can invite 3 friends to go with you to a concert. If you choose Jay, Ted, and Ken, then this is no different from choosing Ted, Ken, and Jay. The order that you choose the three names of your friends is not important. Hence, this is a Combination problem.

We get some redundancies here. Jay, Ted, Ken = Ted, Ken and Jay.

To determine the number of combinations we use the rule:

Practice:

Determine whether each of the following situations is a Combination or Permutation.

1. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from.

2. Voting to allow 10 new members to join a club when there are 25 that would like to join.

Notes Section **10.3 Probability**

- I know the definition of probability and can find the probability of an event.
- I know when events are mutually exclusive with the union of two events
- I can find the probability of the union of two events (mutually exclusive or not)
- I can find the probability of the Intersection of two independent events

What is probability?

Definition of an event:
If S is the sample space of an experiment, then an event is any subset of the sample space:

Events in a sample space:
If an experiment consists of tossing a coin three times and recording the results in order then the sample space is:

$S =$

The event E of showing "exactly two heads" is the subset of S that consists of all outcomes with two heads:

$E =$

The event F of showing "at least two heads" is:

$F =$

The event G of showing "no heads" is:

$G =$

Find the probability:

$$P(E) =$$

A coin is tossed three times and the results are recorded. What is the probability of getting exactly two heads? At least two heads? No heads?

Exactly two heads:

At least two heads:

No heads:

Calculating Probability by Counting:
A five-card poker hand is drawn from a standard deck of 52 cards. What is the probability that all five cards are spades?

A bag contains 20 tennis balls, of which four are defective. If two balls are selected at random from the bag, what is the probability that both are defective?

Probability of the Intersection of Independent Events:

$P(E \cap F) =$

Two dice are rolled how many possible combinations are there?
What is the probability that you will get a 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12?

You have two "Taylor Tokens" what is the probability that you get two rolls more than 10?

Probability of the Union of Events:

Not mutually exclusive:
$P(E \cup F) =$

Mutually exclusive events:
$P(E \cup F) =$

Probability of the Union of Events: Not mutually exclusive
Roll a fair die. The sample space of equally likely simply events is:

Find the probability that you roll a number that is odd or divisible by 3:
Let A = "rolling an odd" and B = "rolling a number divisible by 3"

$P(A) =$ $P(B) =$

$P(A \cap B) =$

$P(odd\ or\ divisible\ by\ 3) =$

Probability of the Union of Events: Mutually exclusive events:
What is the probability of rolling two dice and the sum of the numbers is 7 or 11?

Let A = "sum is 7" and B = "sum is 11".
$S(A) =$ $S(B) =$ $S(E) =$

$P(A) =$ $P(B) =$

$P(A\ or\ B) =$

You try:
What is the probability that you roll a "2" or a "12" to get a candy bar?

What is the probability that both dice turn up the "same number" or the "sum is less than 5"?

A card is drawn randomly from a standard 52-card deck. Find the probability of the given event.

a) The card drawn is a heart

b) The card drawn is either a heart or a spade.

c) The card drawn is not a face card. (This is actually (5c) not (6c))

A poker hand, consisting of five cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains the cards Five hearts

The president of a large company selects six employees to receive a special bonus. He claims that the six employees are chosen randomly from among the 30 employees, of which 19 are women and 11 are men. What is the probability that no woman is chosen?

A monkey is trained to arrange wooden blocks in a straight line. He is then given six blocks showing the letters A, E, H, L, M, T. What is the probability that he will arrange them to spell HAMLET?

Notes Section **10.2 Permutations and Combinations**

- I know the difference between a permutation and a combination.
- I can use the permutation rule to find the arrangement of objects in a specific order
- I can use the combination rule to find the arrangement of objects when order doesn't matter

PERMUTATIONS and COMBINATIONS

If the order doesn't matter, it is a **Combination**.

If the order **does** matter it is a **Permutation**.

Permutations:

A permutation is used when re-arranging the elements of the set creates a new situation.

To determine the number of permutations we use the rule:

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

Permutation:

How many ways could we get 1st, 2nd, and 3rd place winners in a race with the following 4 people? Jay, Sue, Kim, and Bob

${}_4P_3 = 24$ $\quad \frac{4\cdot3\cdot2\cdot1}{1} = 24$ $\quad \underline{4}\cdot\underline{3}\cdot\underline{2}$

${}_4P_2 = 12$ $\quad \frac{4\cdot3\cdot2\cdot1}{2\cdot1} = 12$ $\quad \underline{4}\cdot\underline{3} = 12$

Combinations:

Suppose that you can invite 3 friends to go with you to a concert. If you choose Jay, Ted, and Ken, then this is no different from choosing Ted, Ken, and Jay. The order that you choose the three names of your friends is not important. Hence, this is a Combination problem.

We get some redundancies here. Jay, Ted, Ken = Ted, Ken and Jay. $r!$ we divide out redundancies

To determine the number of combinations we use the rule:

$$C(n,r) = {}_nC_r = \frac{n!}{r!(n-r)!}$$

Practice:

Determine whether each of the following situations is a Combination or Permutation.

1. Determining how many different ways you can elect a Chairman and Co-Chairman of a committee if you have 10 people to choose from. P

2. Voting to allow 10 new members to join a club when there are 25 that would like to join.

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Notes Section **10.3 Probability**

- I know the definition of probability and can find the probability of an event.
- I know when events are mutually exclusive with the union of two events
- I can find the probability of the union of two events (mutually exclusive or not)
- I can find the probability of the Intersection of two independent events

What is probability? The likelihood of an "event" happening.

Definition of an event:
If S is the sample space of an experiment, then an event is any subset of the sample space:

Events in a sample space:
If an experiment consists of tossing a coin three times and recording the results in order then the sample space is:

$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THH, THT\}$

The event E of showing "exactly two heads" is the subset of S that consists of all outcomes with two heads:

$E = \{HHT, HTH, THH\}$

The event F of showing "at least two heads" is:

$F = \{HHH, HHT, HTH, THH\}$

The event G of showing "no heads" is:

$G = \{TTT\}$

Find the probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of elements in } E}{\text{number of elements in } S}$$

A coin is tossed three times and the results are recorded. What is the probability of getting exactly two heads? At least two heads? No heads?

Exactly two heads: $P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

At least two heads: $P(F) = \frac{n(F)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

No heads: $P(G) = \frac{n(G)}{n(S)} = \frac{1}{8}$

Calculating Probability by Counting:

A five-card poker hand is drawn from a standard deck of 52 cards. What is the probability that all five cards are spades?

$P(E) = \frac{n(E)}{n(S)} = \frac{C(13,5)}{C(52,5)} = \frac{1287}{2598960} \approx .05\%$

A bag contains 20 tennis balls, of which four are defective. If two balls are selected at random from the bag, what is the probability that both are defective? $P(E) = \frac{n(E)}{n(S)} = \frac{C(4,2)}{C(20,2)} = \frac{6}{190} \approx 3.2\%$

Probability of the Intersection of Independent Events:

Fundamental

$P(E \cap F) = P(E) \cdot P(F)$

Two dice are rolled how many possible combinations are there?

What is the probability that you will get a 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12? $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

1 2 3 4 5 6 5 4 3 2 1

$2 \text{ or } 12 = \frac{1}{36}$ ~~2.8%~~ $4 \text{ or } 10 = \frac{3}{36} = 8.3\%$ $6 \text{ or } 8 = \frac{5}{36} = 13.9\%$.

$3 \text{ or } 11 = \frac{2}{36} = 5.6\%$ $5 \text{ or } 9 = \frac{4}{36} = 11.1\%$ $7 = \frac{6}{36} = 16.7\%$.

You have two "Taylor Tokens" what is the probability that you get two rolls more than 10?

$\frac{3}{36} \cdot \frac{3}{36} = .69\%$

Probability of the Union of Events:

Not mutually exclusive:

$P(E \cup F) = P(E) + P(F)$

Mutually exclusive events:

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Probability of the Union of Events: Not mutually exclusive

Roll a fair die. The sample space of equally likely simply events is:

$S = \{1, 2, 3, 4, 5, 6\}$

Find the probability that you roll a number that is odd or divisible by 3:

Let A = "rolling an odd" and B = "rolling a number divisible by 3"

$P(A) = \{1, 3, 5\}$ $P(B) = \{3, 6\}$

$P(A \cap B) = \{3\}$

$P(\text{odd or divisible by } 3) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

Probability of the Union of Events: Mutually exclusive events:

What is the probability of rolling two dice and the sum of the numbers is 7 or 11?

Let A = "sum is 7" and B = "sum is 11".

$S(A) =$ (1,6) (6,1) (2,5) (5,2) (4,3) (3,4) $S(B) =$ (5,6), 6(5) $S(E) =$ 36 possibilities

$P(A) = \frac{6}{36}$ $P(B) = \frac{2}{36}$

$P(A \text{ or } B) = \frac{6}{36} + \frac{2}{36} = \frac{2}{9}$

You try:

What is the probability that you roll a "2" or a "12" to get a candy bar?

$\frac{1}{36} + \frac{1}{36} = \frac{1}{18} \approx .05\%$

Sum less than 5 and same number: 2, 4

What is the probability that both dice turn up the "same number" (A) or the "sum is less than 5" (B)?

$P(A) = \frac{6}{36} = \frac{1}{6}$ $\quad P(A) + P(B) - P(A \wedge B) = \frac{1}{6} + \frac{1}{6} - \frac{2}{36} = \frac{5}{18}$

$P(B) = P(1) + P(2) + P(3) + P(4) = \frac{0}{36} + \frac{1}{36} + \frac{2}{36} + \frac{3}{36} = \frac{6}{36} = \frac{1}{6}$ $\quad 27.8\%$

A card is drawn randomly from a standard 52-card deck. Find the probability of the given event.

a) The card drawn is a heart $\quad$ 13 of each suit $\quad \frac{13}{52} = \frac{1}{4}$

b) The card drawn is either a heart or a spade.

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 50\%$

c) The card drawn is not a face card. (~~This is actually (5c) not (6c).~~)

3 face cards of each suit = 12 $\quad \frac{40}{52} = 76.9\%$

A poker hand, consisting of five cards, is dealt from a standard deck of 52 cards. Find the probability that the hand contains the cards Five hearts

$\frac{P(E)}{P(S)} = \frac{C(13,5)}{C(52,5)} = \frac{1287}{2598960} = .000495$

$.05\%$

The president of a large company selects six employees to receive a special bonus. He claims that the six employees are chosen randomly from among the 30 employees, of which 19 are women and 11 are men. What is the probability that no woman is chosen?

$\frac{P(E)}{P(S)} = \frac{C(11,6)}{C(30,6)} = .000778 \approx .08\%$

A monkey is trained to arrange wooden blocks in a straight line. He is then given six blocks showing the letters A, E, H, L, M, T. What is the probability that he will arrange them to spell HAMLET?

$\frac{P(E)}{P(S)} = \frac{1}{{}_6P_6} = \frac{1}{720} \approx .00139$

$.14\%$