Pre-Calculus

Name Key

Hyperbolas

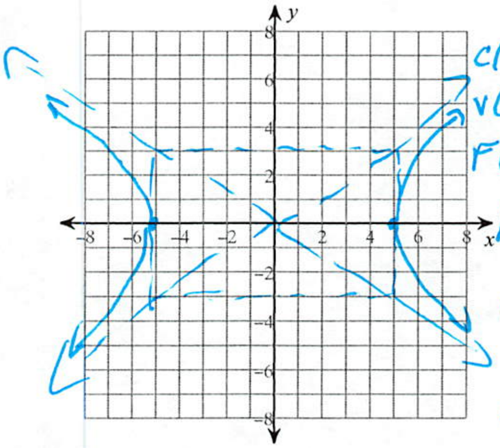
Date________________ Period____

Identify the center, vertices, foci, eccentricity, equation of the asymptotes, and direction of opening. Then sketch the graph.

1) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

$c^2 = 25 + 9$

$c = \sqrt{34}$



$C(0,0)$

$V(-5,0)(5,0)$

$F(-\sqrt{34},0)(\sqrt{34},0)$

Asy: $y = \pm \frac{3}{5}x$

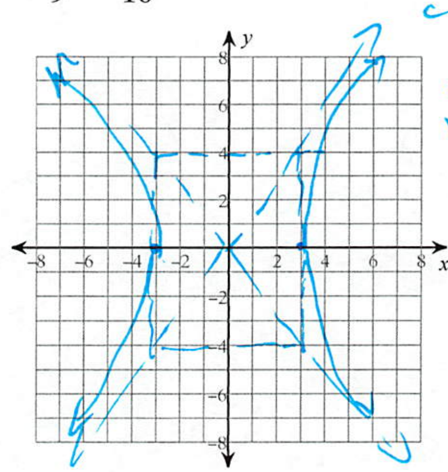
$e = \frac{\sqrt{34}}{5}$

opens: left, right

2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$c^2 = 9 + 16$

$c = 5$



$C(0,0)$

$V(-3,0)(3,0)$

$F(-5,0)(5,0)$

Asy: $y = \pm \frac{4}{3}x$

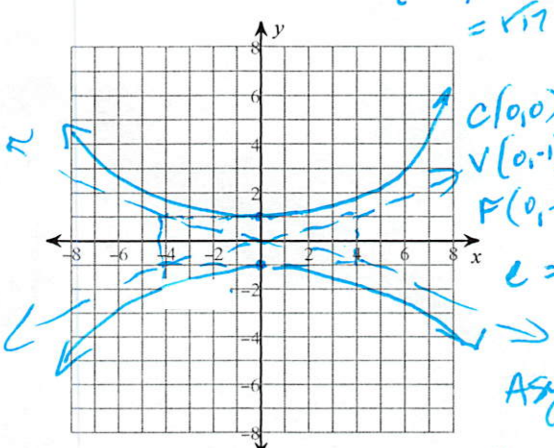
$e = \frac{5}{3}$

opens: left & right

3) $-x^2 + 16y^2 - 16 = 0$

$c^2 = 1 + 16$

$= \sqrt{17}$



$C(0,0)$

$V(0,-1)(0,1)$

$F(0,-\sqrt{17})(0,\sqrt{17})$

$e = \sqrt{17}$

Asy: $y = \pm \frac{1}{4}x$

direction: up & down

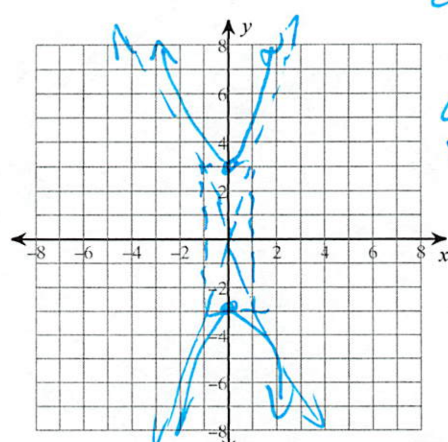
$16y^2 - x^2 = 16$

$\frac{y^2}{1} - \frac{x^2}{16} = 1$

4) $-9x^2 + y^2 - 9 = 0$

$c^2 = 9 + 1$

$c = \sqrt{10}$



$C(0,0)$

$V(0,-3)(0,3)$

$F(0,-\sqrt{10})(0,\sqrt{10})$

$e = \frac{\sqrt{10}}{3}$

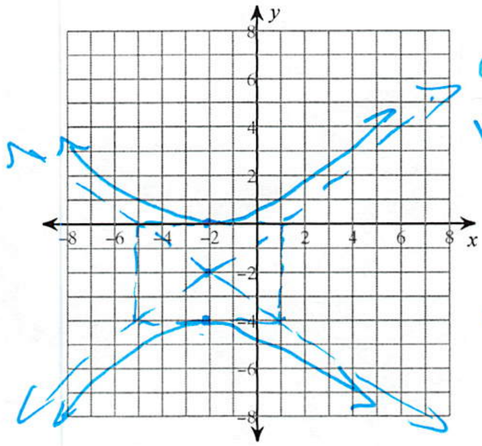
Asy: $y = \pm 3x$

$y^2 - 9x^2 = 9$

$\frac{y^2}{9} - \frac{x^2}{1} = 1$

5) $\dfrac{(y+2)^2}{4} - \dfrac{(x+2)^2}{9} = 1$

$c^2 = 4+9$
$c = \sqrt{13}$



$C(-2,-2)$

$V(-2,0)\,(-2,-4)$

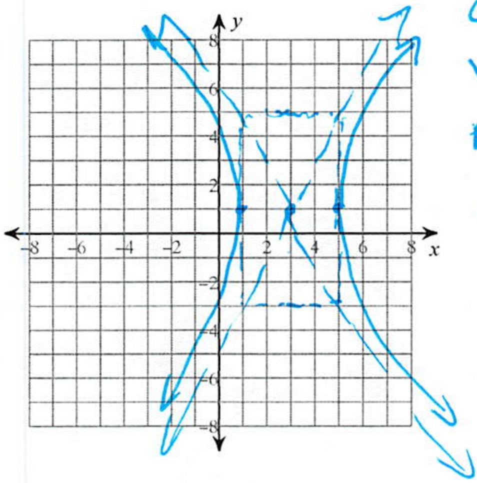
$F(-2,-2-\sqrt{13})\,(-2,-2+\sqrt{13})$

Asy: $y = \pm\frac{2}{3}(x+2) - 2$

$e = \dfrac{\sqrt{13}}{2}$ Direction: up ; down

6) $\dfrac{(x-3)^2}{4} - \dfrac{(y-1)^2}{16} = 1$

$c^2 = 4+16$, $c = \sqrt{20} = 2\sqrt{5}$



$C(3,1)$

$V(1,1)\,(5,1)$

$F(3-2\sqrt{5},1),(3+2\sqrt{5},1)$

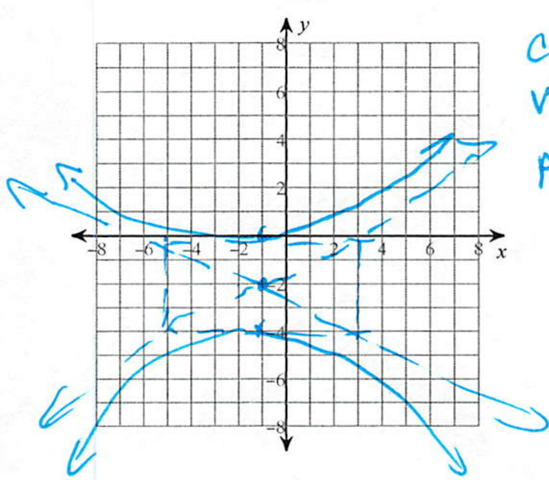
$e = \dfrac{2\sqrt{5}}{2} = \sqrt{5}$

Asy: $y = \pm 2(x-3)+1$

Direction: left ; right

7) $\dfrac{(y+2)^2}{4} - \dfrac{(x+1)^2}{16} = 1$

$c^2 = 4+16$, $c = \sqrt{20} = 2\sqrt{5}$



$C(-1,-2)$

$V(-1,0)\,(-1,-4)$

$F(-1,-2-2\sqrt{5})(-1,-2+2\sqrt{5})$

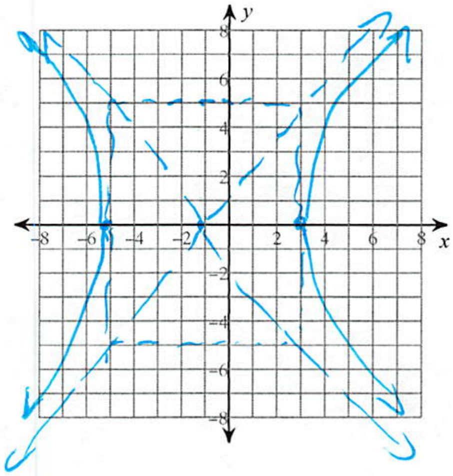
$e = \dfrac{2\sqrt{5}}{2} = \sqrt{5}$

Asy: $y = \pm\frac{1}{2}(x+1) - 2$

Direction: up ; down

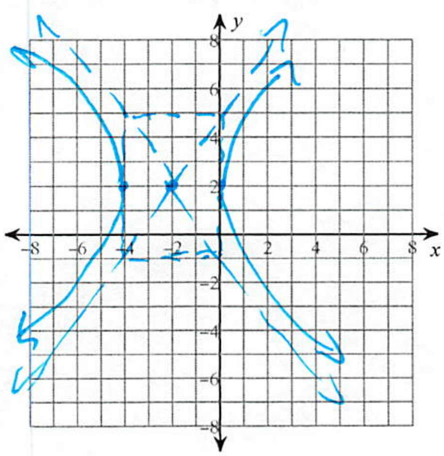
8) $\dfrac{(x+1)^2}{16} - \dfrac{y^2}{25} = 1$

$c^2 = 16 + 25$
$c = \sqrt{41}$



$C(-1,0)$
$V(3,0)(-5,0)$
$F(-1+\sqrt{41}, 0)(-1-\sqrt{41}, 0)$
$e = \dfrac{\sqrt{41}}{4}$
asy: $y = \pm \frac{5}{4}(x+1)$
Direction: left & right

9) $9x^2 - 4y^2 + 36x + 16y - 16 = 0$



$9x^2 + 36x \quad -4y^2 + 16y \quad = 16$
$9(x^2 + 4x + 4) - 4(y^2 - 4y + 4) = 16 \; {+36} \; {-16}$
$9(x+2)^2 - 4(y-2)^2 = 36$
$\dfrac{(x+2)^2}{4} - \dfrac{(y-2)^2}{9} = 1$

$C(-2,2)$ $\qquad c^2 = 4 + 9, \; c = \sqrt{13}$
$V(-4,2),(0,2)$
$F(-2-\sqrt{13},2)(-2+\sqrt{13},2)$ $\qquad$ Direction: left & right
$e = \dfrac{\sqrt{13}}{2}$ $\qquad$ asy: $y = \pm \frac{3}{2}(x+2) + 2$

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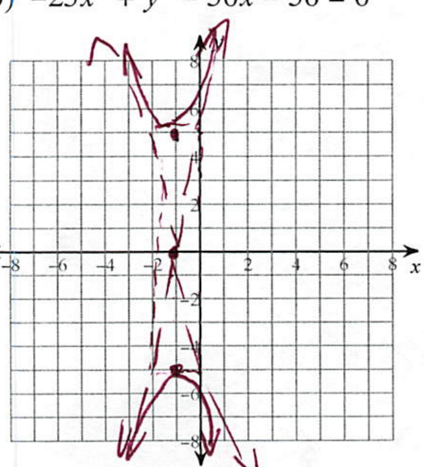
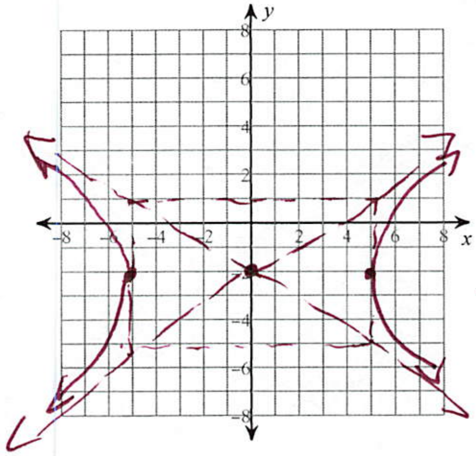
10) $-25x^2 + y^2 - 50x - 50 = 0$



$y^2 - 25x^2 - 50x = 50$
$y^2 - 25(x^2 + 2x + 1) = 50 \; {-25}$
$y^2 - 25(x+1)^2 = 25$
$\dfrac{y^2}{25} - \dfrac{(x+1)^2}{1} = 1$ $\qquad c^2 = 25 + 1$, $c = \sqrt{26}$

$C(-1,0)$
$V(-1,5),(-1,-5)$
$F(-1,\sqrt{26}),(-1,-\sqrt{26})$
$e = \dfrac{\sqrt{26}}{5}$
asym: $y = \pm 5(x+1)$
Direction: up & down

11) $9x^2 - 25y^2 - 100y - 325 = 0$



$9x^2 - 25y^2 - 100y = 325$

$9x^2 - 25(y^2 + 4y + 4) = 325 - 100$

$9x^2 - 25(y+2)^2 = 225$

$\frac{x^2}{25} - \frac{(y+2)^2}{9} = 1$ $c^2 = 25 + 9$, $c = \sqrt{34}$

$C(0, -2)$

$V(5, -2)(-5, -2)$

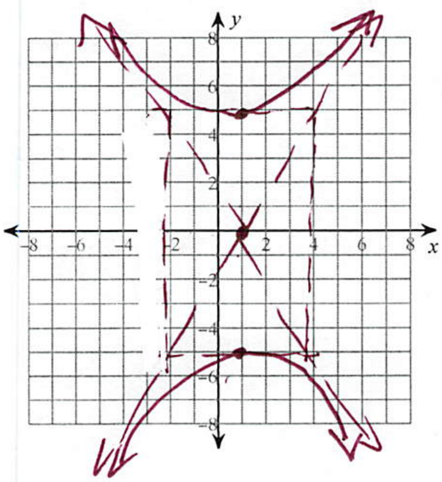
$F(\sqrt{34}, -2)(-\sqrt{34}, -2)$

asy: $y = \pm \frac{3}{5}(x) - 2$

$e = \frac{\sqrt{34}}{5}$

Direction: left; right

12) $-25x^2 + 9y^2 + 50x - 250 = 0$



$9y^2 - 25x^2 + 50x = 250$

$9y^2 - 25(x^2 - 2x + 1) = 250 - 25$

$9y^2 - 25(x-1)^2 = 225$

$\frac{y^2}{25} - \frac{(x-1)^2}{9} = 1$ $c^2 = 25 + 9$, $c = \sqrt{34}$

$C(1, 0)$

$V(1, 5)(1, -5)$

$F(1, \sqrt{34})(1, -\sqrt{34})$

asy: $y = \pm \frac{5}{3}(x-1)$

$e = \frac{\sqrt{34}}{5}$

Use the information provided to write the standard form equation of each hyperbola.

13) Vertices: $(-1, 9)$, $(-15, 9)$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Endpoints of Conjugate Axis: $(-8, 18)$
$(-8, 0)$

$2a = 14$ $a = 7$ $C(-8, 9)$
$2b = 18$ $b = 9$

$\boxed{\frac{(x+8)^2}{49} - \frac{(y-9)^2}{81} = 1}$

14) Vertices: $(2, 4)$, $(-18, 4)$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Foci: $(-8 + 10\sqrt{2}, 4)$, $(-8 - 10\sqrt{2}, 4)$

$C(-8, 4)$ $a = 10$ $c = 10\sqrt{2}$

$(10\sqrt{2})^2 = 10^2 + b^2$

$b^2 = 100$

$\boxed{\frac{(x+8)^2}{100} - \frac{(y-4)^2}{100} = 1}$

15) Vertices: $(3, 17)$, $(3, -3)$

Asymptotes: $y = \frac{5}{3}x + 2$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$y = -\frac{5}{3}x + 12$

$a = 10$

$(h, k) = (3, 7)$

$\frac{a}{b} = \frac{5}{3} = \frac{10}{b}$ $b = 6$

$\boxed{\frac{(y-7)^2}{100} - \frac{(x-3)^2}{36} = 1}$

16) Foci: $(1 + 2\sqrt{61}, 6)$, $(1 - 2\sqrt{61}, 6)$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Endpoints of Conjugate Axis: $(1, 16)$
$(1, -4)$

$c = 2\sqrt{61}$

$(h, k) = (1, 6)$

$b = 10$

$(2\sqrt{61})^2 = a^2 + 10^2$

$a^2 = 144$

$\boxed{\frac{(x-1)^2}{144} - \frac{(y-6)^2}{100} = 1}$

17) Center at $(-10, 7)$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Vertex at $(3, 7)$

Eccentricity $= \frac{5\sqrt{10}}{13}$

$\boxed{\frac{(x+10)^2}{169} - \frac{(y-7)^2}{81} = 1}$

$a = 13$ $C(-10, 7)$

$c = 5\sqrt{10}$ $c^2 = a^2 + b^2$

$(5\sqrt{10})^2 = 13^2 + b^2$; $b^2 = 81$

18) Center at $(6, 4)$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transverse axis is vertical and 18 units long
Congujate axis is 20 units long

$2a = 18$
$a = 9$

$2b = 20$
$b = 10$

$\boxed{\frac{(y-4)^2}{81} - \frac{(x-6)^2}{100} = 1}$