Pre-Calculus Name Key ID: 1

Review Final

Date _____ Period ____

Chapter 1/2 Linear Equations and Inequalities

Write the slope-intercept form of the equation of the line through the given points.

1) through: $(-3, -5)$ and $(-4, -1)$

$m = \frac{-5+1}{-3+4} = \frac{-4}{1}$

$y = mx + b$

$-1 = (4)(-4) + b$

$-1 = 16 + b$

$-17 = b$

$\boxed{y = -4x - 17}$

Solve each inequality.

2) $8 \le -1 - 9m < 89$

$+1 \quad +1 \quad +1$

$\frac{9}{-9} \le \frac{-9m}{-9} < \frac{90}{-9}$

$-1 \ge m > -10$

$\boxed{-10 < m \le -1}$

3) $-7|9 - 5m| + 3 > -95$

$-7|9-5m| > -98$

$|9-5m| < 14$

$9 - 5m < 14$; $9 - 5m > -14$

$-5m < 5$; $-5m > -23$

$m > -1$; $m < 23/5$

$\boxed{-1 < m < \frac{23}{5}}$

Chapter 3 Functions

Evaluate each function.

4) $h(t) = -2t^2 + 2 + t$; Find $h(5)$

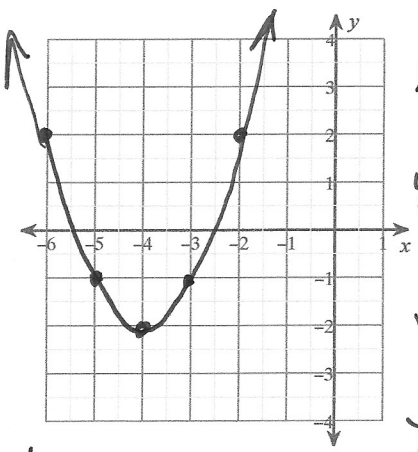
$h(5) = -2(5)^2 + 2 + 5$

$= -43$

Identify the domain and range of each. Then sketch the graph.

5) $f(x) = x^2 + 8x + 14$

$x = \frac{-b}{2a}$



$x = -\frac{8}{2(1)} = -4$

$y = (-4)^2 + 8(-4) + 14$

$= -2$

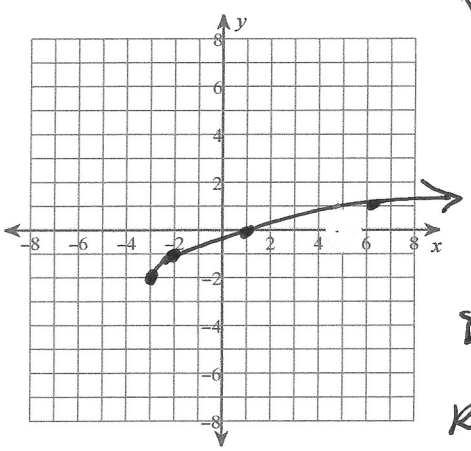
$V(-4, -2)$

D: $(-\infty, \infty)$

R: $[-2, \infty)$

x	y
-3	-1
-2	2

6) $y = -2 + \sqrt{x+3}$



$V(-3, -2)$

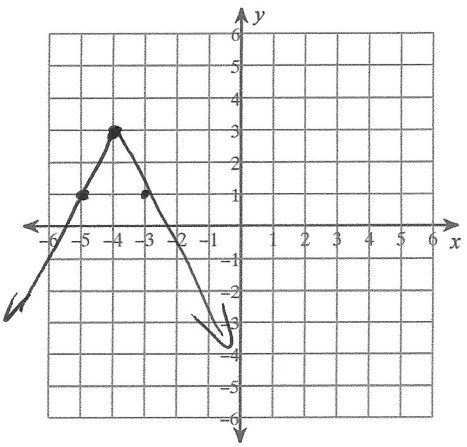
x	y
-2	-1
1	0
6	1

D: $[-3, \infty)$

R: $[-2, \infty)$

Graph the equation and Identify the domain and range.

7) $y = -2|x+4| + 3$



V (-4, 3)

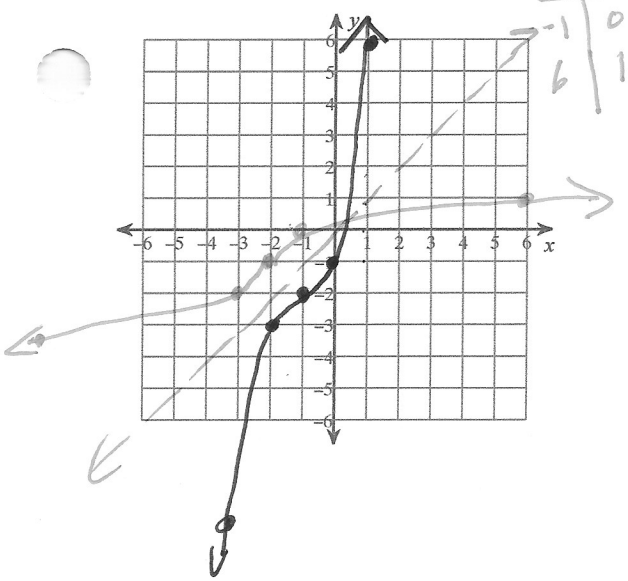
D: $(-\infty, \infty)$

R: $(-\infty, 3]$

Find the inverse of each function. Then graph the function and its inverse.

8) $f(x) = \sqrt[3]{x+2} - 1$ V (-2, -1)

x	y
-1	0
6	1



$y = \sqrt[3]{x+2} - 1$

$x = \sqrt[3]{y+2} - 1$

$x+1 = \sqrt[3]{y+2}$

$(x+1)^3 = y+2$

$y = (x+1)^3 - 2$ inverse

V (-1, -2)

x	y
0	-1
1	6

Perform the indicated operation.

9) $f(x) = x^3 + 1$
$g(x) = 2x + 5$
Find $f(x) + g(x)$

$x^3 + 1 + 2x + 5$

$x^3 + 2x + 6$

10) $g(n) = 4n + 5$
$f(n) = n^2 + 3$
Find $g(f(n))$

$4(n^2+3) + 5$

$4n^2 + 12 + 5$

$4n^2 + 17$

Chapter 4 Polynomials

Use Long Division to Divide.

11) $\dfrac{x^4 + 11x^3 + 19x^2 + 19x + 80}{x^2 + 9}$

$x^2 + 11x + 10 + \dfrac{-80x - 10}{x^2+9}$

$x^2+9 \;\big|\; x^4 + 11x^3 + 19x^2 + 19x + 80$

$-(x^4 \quad + 9x^2)$

$11x^3 + 10x^2 + 19x$

$-(11x^3 \quad + 99x)$

$10x^2 - 80x + 80$

$-(10x^2 \quad + 90)$

$-80x - 10$

Use Synthetic Division to Divide.

12) $(x^5 + 4x^4 - 70x^3 + 56x^2 + 18x + 36) \div (x - 6)$

6	1	4	−70	56	18	36
		6	60	−60	−24	−36
	1	10	−10	−4	−6	0

$x^4 + 10x^3 - 10x^2 - 4x - 6$

Find all zeros. One zero has been given.

13) $f(x) = x^4 + 3x^2 - 7x^2 - 27x - 18;\ -2$

−2	1	3	−7	−27	−18
		−2	−2	18	18
	1	1	−9	−9	0

−3	1	1	−9	−9
		−3	6	9
	1	−2	−3	0

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$-2, -3, 3, -1$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

14) $-3,\ -1,\ 1,\ \sqrt{5}$

$(x+3)(x+1)(x-1)(x-\sqrt{5})(x+\sqrt{5})$

$(x+3)(x^2-1)(x^2-5)$

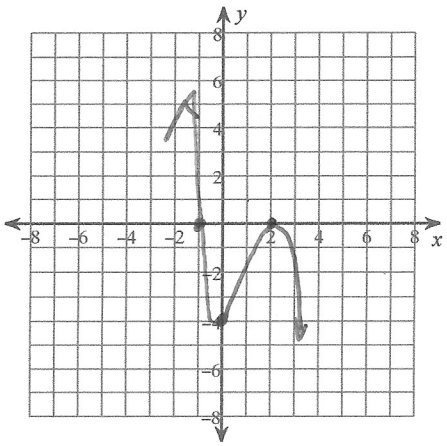
$(x+3)(x^4-6x^2+5)$

$x^5 - 6x^3 + 5x + 3x^4 - 18x^2 + 15$

$x^5 + 3x^4 - 6x^3 - 18x^2 + 5x + 15$

Sketch the graph of each function. State the number of real zeros and the end behavior.

15) $f(x) = -x^3 + 3x^2 - 4$



y-int $(0, -4)$

-1 | -1 | 3 | 0 | -4

	1	-4	4
-1	4	-4	0

$-x^2 + 4x - 4 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

Zero's: $-1, 2$ (mult 2)

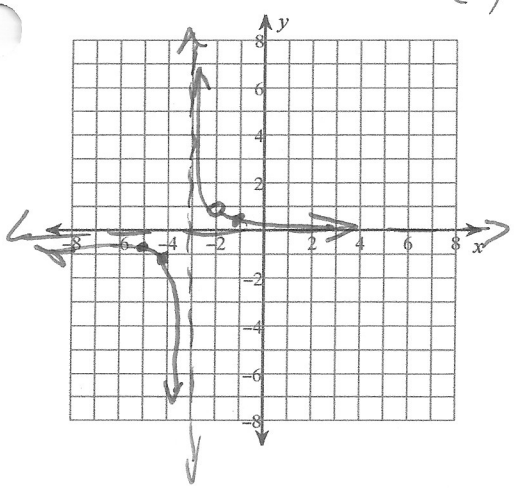
End Behavior

$x \to +\infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to +\infty$

Identify the vertical asymptote(s), holes, horizontal asymptote, and domain. Then sketch the graph.

16) $f(x) = \dfrac{x+2}{x^2+5x+6}$ $= \dfrac{x+2}{(x+2)(x+3)} = \dfrac{1}{x+3}$ (hole at $x = -2$)



V.A. $x = -3$

H.A. $y = 0$

x	y	
-5	$-1/2$	
-4	-1	
-2	1	hole
-1	$1/2$	

Solve each equation. Remember to check for extraneous solutions.

17) $\dfrac{5}{4x} + \dfrac{1}{4} = \dfrac{1}{2}$

$\dfrac{5}{4x} + \dfrac{x}{4x} = \dfrac{2x}{4x}$

$5 + x = 2x$

$x = 5$

18) $\dfrac{n-2}{2n^2} + \dfrac{n-5}{4n^2} = \dfrac{n-1}{4n^2}$

$\dfrac{2(n-2)}{4n^2} + \dfrac{(n-5)}{4n^2} = \dfrac{(n-1)}{4n^2}$

$2n - 4 + n - 5 = n - 1$

$3n - 9 = n - 1$

$2n = 8$

$n = 4$

Use a calculator to approximate each to the nearest thousandth.

19) $\log_4 7$ $= \frac{\log 7}{\log 4} = 1.404$

Identify the domain and range of each.

20) $y = \log_4 (4x - 3) + 4$

$4x - 3 > 0$
$4x > 3$
$x > 3/4$

D: $(\frac{3}{4}, \infty)$
R: $(-\infty, \infty)$

Condense each expression to a single logarithm.

21) $3\log_8 x + 6\log_8 y + 3\log_8 z$

$= \log_8 x^3 + \log_8 y^6 + \log_8 z^3$
$= \log_8 x^3 y^6 z^3$

22) $20\log_5 c + 20\log_5 a - 4\log_5 b$

$= \log_5 c^{20} + \log_5 a^{20} - \log_5 b^4$
$= \log_5 \frac{c^{20} a^{20}}{b^4}$

Expand each logarithm.

23) $\log_3 \frac{zx^4}{y^2}$

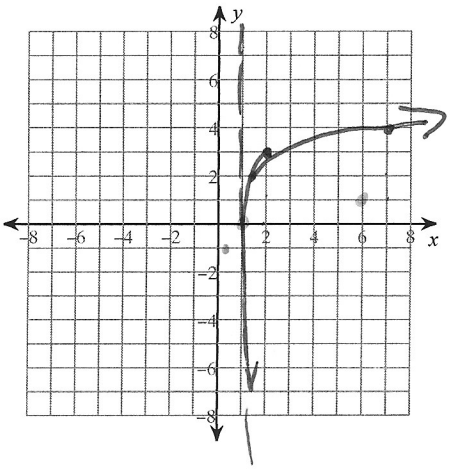
$= \log_3 zx^4 - \log_3 y^2$
$= \log_3 z + \log_3 x^4 - 2\log_3 y$
$\log_3 z + 4\log_3 x - 2\log_3 y$

24) $\log_4 (xy^2 \cdot z)^3$

$\log_4 x^3 y^6 z^3$
$\log_4 x^3 + \log_4 y^6 + \log_4 z^3$
$3\log_4 x + 6\log_4 y + 3\log_4 z$

Identify the domain and range of each. Then sketch the graph.

25) $y = \log_6 (x - 1) + 3$



D: $(1, \infty)$
R: $(-\infty, \infty)$

$\log_6 x$

x	y
1/6	-1
1	0
6	1

→ 1 right
↑ up 3

Solve each equation. Round your answers to the nearest hundreth

26) $8^{n-1} - 10 = 69$

$8^{n-1} = 79$

$n - 1 = \log_8 79$

$n = \frac{\log 79}{\log 8} + 1$

≈ 3.10

27) $-0.6 \cdot 20^{x-3} = -54$

$20^{x-3} = 90$

$x - 3 = \log_{20} 90$

$x = \frac{\log 90}{\log 20} + 3$

$x \approx 4.50$

Solve each equation.

28) $\log_4 (x+3) - \log_4 (x-2) = 2$

$\log_4 \frac{x+3}{x-2} = 2$

$\frac{x+3}{x-2} = 4^2$

$x + 3 = 16(x-2)$

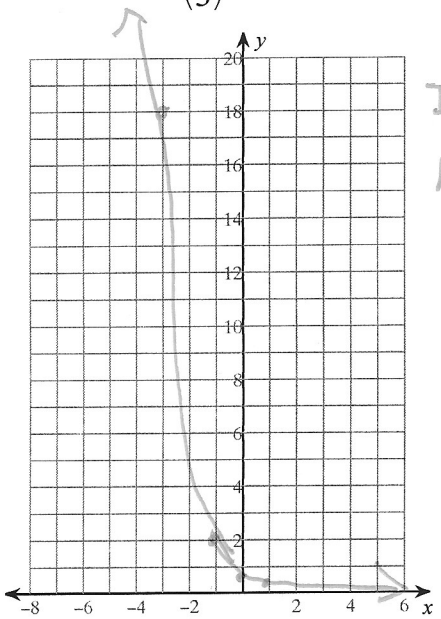
$x + 3 = 16x - 32$

$35 = 15x$

$x = \frac{7}{3}$

Sketch the graph of each function. Identify the Domain and Range and the Asymptotes.

29) $f(x) = 2 \cdot \left(\frac{1}{3}\right)^{x+1}$

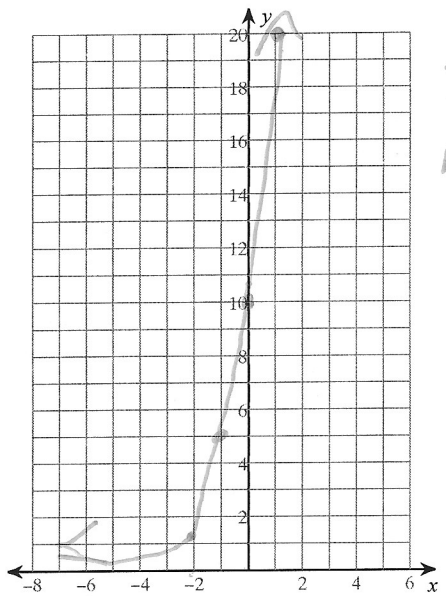


x	y
-1	2
0	2/3
1	2/9
-3	18

D: $(-\infty, \infty)$

R: $(0, \infty)$

30) $f(x) = 5 \cdot 2^{x+1}$



D: $(-\infty, \infty)$

R: $(0, \infty)$

x	y
-1	5
0	10
1	20
-2	5/4

Chapter 6 Systems of Equations and Inequalities

Solve each system by substitution or elimination.

31) $-12x - 3y = 12$
$-6x - 9y = -24$

-3 | $36x + 9y = -36$
$-6x - 9y = -24$

$30x = -60$

$x = -2$

$-6(-2) - 9y = -24$

$-9y = -36$

$y = 4$

$(-2, 4)$

32) $3x + y = -13$
$-4x - 4y = -4$

$3(-7) + y = -13$

$y = 8$

$y = -13 - 3x$

$-4x - 4(-13 - 3x) = -4$

$-4x + 52 + 12x = -4$

$8x = -56$

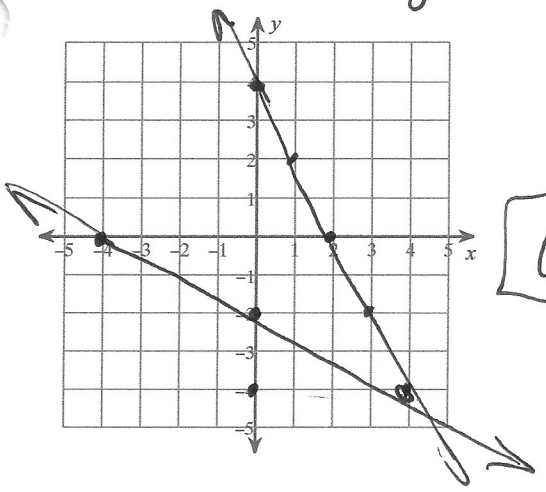
$x = -7$

$(-7, 8)$

Solve each system by graphing.

33) $2x + y = 4$ $x = 2$ $y = 4$

$x + 2y = -4$ $x = -4$ $y = -2$



(4, -4)

Solve each system by elimination.

34) $2x + 4y - z = -20$

$-2x - y + 2z = 15$

$3x - y + 2z = 5$

$2 \Rightarrow 4x + 8y - 2z = -40 \Rightarrow 4x + 8y - 2z = -40$

$-2x - y + 2z = 15$

$3x - y + 2z = 5$

$2x + 7y = -25$

$-7x - 7y = 35$

$-5x = 10$

$x = -2$

$(7x + 7y = -35)(-1)$

$7(-2) + 7y = -35$

$7y = -21$

$y = -3$

$2(-2) + 4(-3) - z = -20$

$-4 - 12 - z = -20$

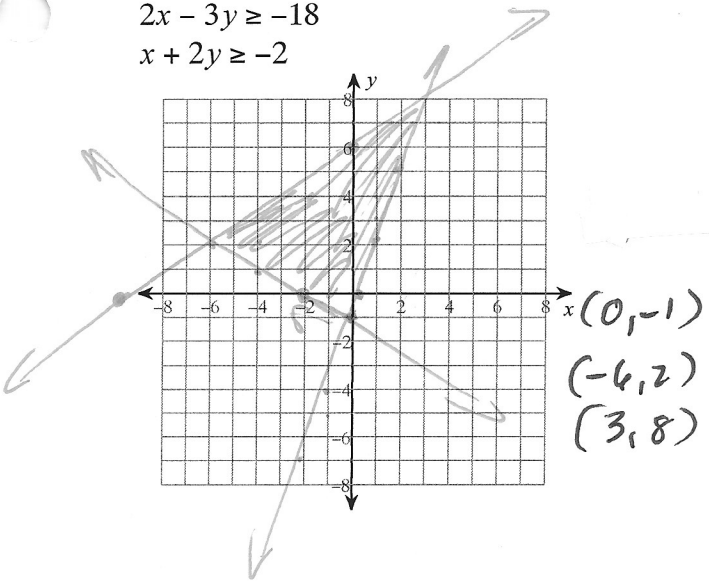
$-z = -4$

$z = 4$

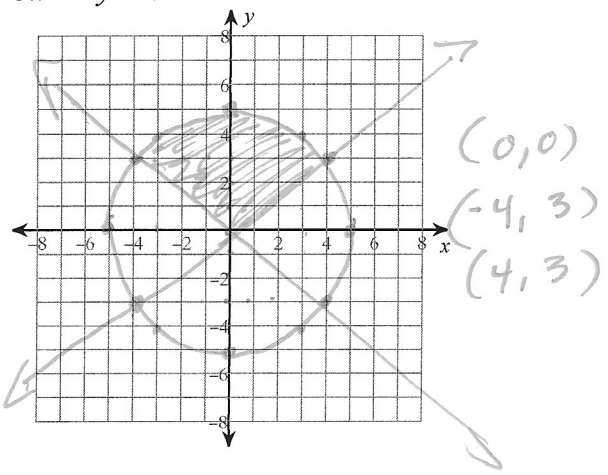
(-2, -3, 4)

Sketch the solution to each system of inequalities. If the region is bounded state the coordinates of the vertices.

35) $3x - y \le 1$
$2x - 3y \ge -18$
$x + 2y \ge -2$



36) $x^2 + y^2 \le 25$
$y \le \frac{3}{4}x$
$3x + 4y \ge 0$



Chapter 7 Matrices

Simplify. Write "undefined" for expressions that are undefined.

37) $\begin{bmatrix} 1 & 0 & 6 \\ -4 & -6 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 2 & -2 \\ 4 & 1 \end{bmatrix}$ $\begin{matrix} 1 & -4 \\ 0 & -6 \\ 6 & -1 \end{matrix}$ $= \begin{bmatrix} 22 & 3 \\ -8 & 23 \end{bmatrix}$

Use your calculator to evaluate the determinant.

38) $\begin{vmatrix} 1 & -5 & 0 \\ -1 & -4 & 4 \\ -2 & -2 & 2 \end{vmatrix}$ $= 30$

Use Cramer's Rule to solve each system.

39) $-6x + 2y = 10$
$-4x - 4y = -4$

$D = \begin{vmatrix} -6 & 2 \\ -4 & -4 \end{vmatrix} = 32$

$D_x = \begin{vmatrix} 10 & 2 \\ -4 & -4 \end{vmatrix} = -32$

$D_y = \begin{vmatrix} -6 & 10 \\ -4 & -4 \end{vmatrix} = 64$

$x = \frac{D_x}{D} = \frac{-32}{32} = -1$

$y = \frac{D_y}{D} = \frac{64}{32} = 2$

$(-1, 2)$

Write the matrix equation for the system. Then solve using inverse matrices.

40) $10x + 2y = -6$
$-7x - 3y = 1$

$\begin{vmatrix} 10 & 2 \\ -7 & -3 \end{vmatrix} = -16$

$A^{-1} = -\frac{1}{16}\begin{vmatrix} -3 & -2 \\ 7 & 10 \end{vmatrix}$

$(-1, 2)$

$\begin{bmatrix} 10 & 2 \\ -7 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$ (A, B)

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B$

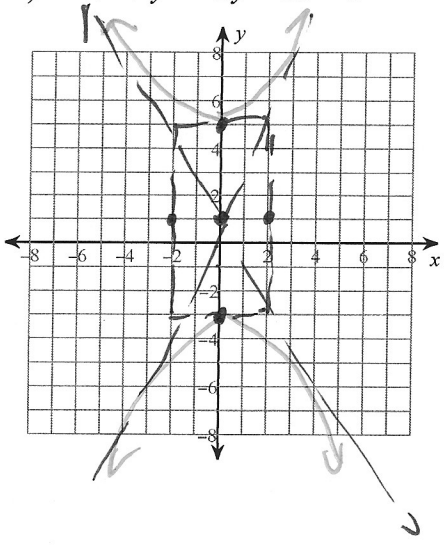
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$-\frac{1}{16}\begin{vmatrix} -3 & -2 \\ 7 & 10 \end{vmatrix} \cdot \begin{bmatrix} -6 \\ 1 \end{bmatrix} \cdot \begin{matrix} \cdot 3 \\ -2 \end{matrix} \quad \begin{matrix} 7 \\ 10 \end{matrix} = -\frac{1}{16}\begin{bmatrix} 16 \\ -32 \end{bmatrix}$

Chapter 8 Conics

Classify each conic section, write its equation in standard form, and sketch its graph. For parabolas, identify the vertex and focus. For circles, identify the center. For ellipses and hyperbolas identify the center, vertices, and foci.

41) $-4x^2 + y^2 - 2y - 15 = 0$



$y^2 - 2y \quad -4x^2 \quad = 15 \quad +1$

$y^2 - 2y + 1 \quad -4x^2$

$(y-1)^2 - 4x^2 = 16$

$\frac{(y-1)^2}{16} - \frac{x^2}{4} = 1$ Hyperbola

$c^2 = 16 + 4$

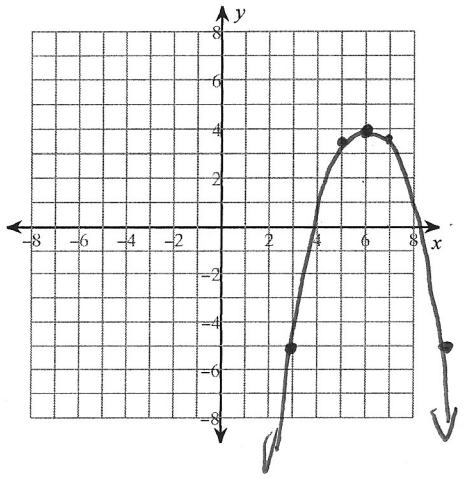
$c = \sqrt{20} = 2\sqrt{5}$

$C(0, 1)$

$V(0, 5), (0, -3)$

$F(0, 1 \pm 2\sqrt{5})$

42) $x^2 - 12x + y + 32 = 0$



$x^2 - 12x = -y - 32$

$x^2 - 12x + 36 = -y - 32 + 36$

$(x-6)^2 = -(y-4)$

Parabola

$v(6,4)$

$4p = -1$

$p = -\frac{1}{4}$

goes down

$3\frac{3}{4}$ | -5

focus $(6, 3\frac{3}{4})$

focal length $= 1$

Chapter 9 Sequences and Series

Given a term in an arithmetic sequence and the common difference find the nth term and the term named in the problem.

43) $a_{23} = 48,\ d = 2$

Find a_{35}

$a_n = a_1 + (n-1)d$

$a_{23} = 48 = a_1 + (23-1)2$

$48 = a + 44$

$4 = a$

$a_n = 4 + (n-1)2$

$a_{35} = 4 + 34(2)$

$a_{35} = 72$

Determine if the sequence is geometric.

If it is, find the common ratio, the nth term and the term named in the problem.

44) $-2,\ 4,\ -8,\ 16, \ldots$

Find a_9

$r = -2$

$a_n = -2(-2)^{n-1}$

$a_9 = -2(-2)^8 = -512$

Evaluate each arithmetic series described.

45) $\sum_{m=5}^{9} (5m + 3)$

$n = 5$

$a_5 = 28$

$a_9 = 48$

$S_n = \frac{n}{2}(a + a_n)$

$S_n = \frac{5}{2}(28 + 48) = 190$

Evaluate each geometric series described.

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

46) $\sum_{n=1}^{10} 2 \cdot 4^{n-1}$

$a_1 = 2$
$r = 4$
$n = 10$

$$S_n = 2\left(\frac{1-4^{10}}{1-4}\right)$$

$$= 699{,}050$$

Evaluate each infinite geometric series described.

47) $-3 - \frac{3}{2} - \frac{3}{4} - \frac{3}{8} \ldots$

$r = \frac{1}{2}$

$$S_n = \frac{a}{1-r} = \frac{-3}{1-\frac{1}{2}} = \frac{-3}{\frac{1}{2}} = -6$$

Expand completely.

48) $(4x + y)^3$

1 3 3 1
$(4x)^3$ $(4x)^2$ $4x$ y y^2 y^3

$$64x^3 + 48x^2y + 12xy^2 + y^3$$

Find each term described.

49) 4th term in expansion of $(2y + x^2)^6$

$y^{\square}$ $s^{\square}$ $c^{\square}$ $7^{\square}$
$\binom{6}{3}$
20
$(2y)^3$
$(x^2)^3$

$$= 20(8)y^3x^6$$

$$160y^3x^6$$

Trig Chapter 2/3
Find the exact value of each trigonometric function.

50) $\sin\frac{5\pi}{4} - \cos\frac{5\pi}{6}$

$$-\frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2}$$

$$\frac{-\sqrt{2}+\sqrt{3}}{2}$$

51) $\sec\frac{\pi}{4}\cos\frac{5\pi}{4}$

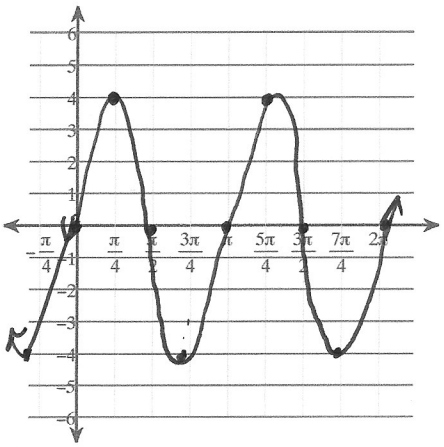
$$\sqrt{2}\left(\frac{-\sqrt{2}}{2}\right) = -1$$

Using radians, find the amplitude and period of each function. Then graph.

52) $y = 4\sin 2\theta + 1$

$a = 4$

$p = \frac{2\pi}{2} = \pi$



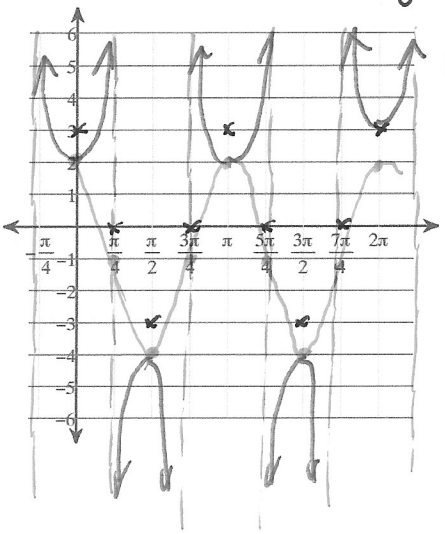
53) $y = -1 + 3\sec 2\theta$

$y = 3\cos 2\theta - 1$

$a = 3$

$p = \frac{2\pi}{2} = \pi$

↓ Down 1



Trig Chapter 4

Use the angle sum or difference identity to find the exact value of each.

54) $\sin\frac{13\pi}{12}$ (Hint: $\frac{13\pi}{12} = \frac{5\pi}{6} + \frac{\pi}{4}$) $\sin(x + y) = \sin x \cdot \cos y + \sin y \cdot \cos x$

$\sin\frac{13\pi}{12} = \sin\left(\frac{5\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{5\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{5\pi}{6}$

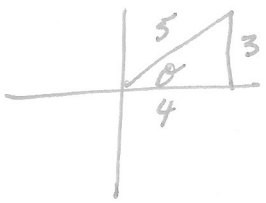
$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

Use a double-angle identity to find the exact value of each expression.

55) $\cos\theta = \frac{4}{5}$ and $0 < \theta < \frac{\pi}{2}$

Find $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\cdot\theta}$



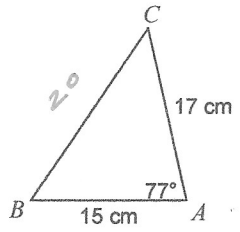
$\tan\theta = \frac{3}{4}$

$\tan 2\theta = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{3/2}{7/16} = \boxed{\frac{24}{7}}$

Use the law of Cosines to solve each triangle. Round your answers to the nearest tenth.

$a^2 = b^2 + c^2 - 2bc \cdot \cos \cdot A$

56)



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 15^2 + 17^2 - 2(15)(17)\cos 77°$

$a \approx 20$

$\frac{\sin 77}{20} = \frac{\sin C}{15}$

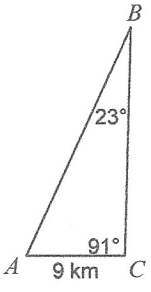
$B = 56°$

$C \approx 47°$

Use the law of Sines to solve each triangle. Round your answers to the nearest tenth.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

57)



$\frac{\sin 23}{9} = \frac{\sin 91}{c}$

$c \approx 23$

$\angle A = 66°$

$a \approx 21$

$\frac{\sin 66}{a} = \frac{\sin 23}{9}$

Verify the following trig identities:

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

58) $\frac{1 - \cos^2 \cdot x}{\cos x} = \tan x \cdot \sin x$

$\hookrightarrow = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \cdot \sin x = \tan x \cdot \sin x$

Trig Chapter 5/6

Simplify.

59) $(6 + 2i)(-3 - 7i)$

$-18 - 42i - 6i - 14i^2$

$+14$

$-4 - 48i$

Simplify.

60) $\dfrac{6}{-1-10i}$ $\dfrac{(-1+10i)}{(-1+10i)}$ $\dfrac{-6+60i}{(-1)^2-(10i)^2} = \dfrac{-6+60i}{1--100} = \boxed{\dfrac{-6+60i}{101}}$

Convert each of the following points from polar to rectangular coordinates or rectangular to polar coordinates:

$x = r\cos\theta,\ y = r\sin\theta$ $\qquad$ $x^2 + y^2 = r^2,\ \tan\theta = \dfrac{y}{x}$

61) $x = r\cos\theta,\ y = r\sin\theta$

$\left(3, \dfrac{\pi}{3}\right)$

$x = 3\cos\frac{\pi}{3} = \frac{3}{2}$

$y = 3\sin\frac{\pi}{3} = \frac{3\sqrt{3}}{2}$

$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

62) $x^2 + y^2 = r^2,\ \tan\theta = \dfrac{y}{x}$

$(2, -2)$

$r^2 = 2^2 + (-2)^2$

$r = 2\sqrt{2}$

$\tan\theta = -\frac{2}{2}$

$\theta = -\frac{\pi}{4}$

$\left(2\sqrt{2}, -\frac{\pi}{4}\right)$

Convert each of the rectangular equations to polar form. (Solve for r).

63) $x^2 + y^2 - 9 = 0$

$x^2 + y^2 = 9$

$r^2 = 9$

$\boxed{r = 3}$

Convert each of the polar equations to rectangular form.

64) $r = 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

$\boxed{x^2 + y^2 - 4y = 0}$