Name: Key

MATH 1030: Unit 4 Study Guide

Chapter 7 Probability: Living with the Odds

7A Fundamentals of Probability

Theoretical Probabilities:

Use the theoretical method to determine the probability of the following outcomes and events. State any assumptions that you make.

1- A randomly selected person was born on a Wednesday. $1/7$

Event Not Occurring:

Determine the probability of the following events. State any assumptions that you use.

2- What is the probability of not tossing three tails with three fair coins?

only 1 way to get 3 tails $\frac{7}{8}$

Odds:

Find both the odds for and the odds against the following events.

3- Flipping two fair coins and getting two tails.

odds for $\frac{1}{2}$ odds against $\frac{2}{1}$

Computing Probabilities:

Decide which method (theoretical, relative frequency, or subjective) is appropriate, and compute or estimate the following probabilities.

4- Waiting longer than 15 minutes when you arrive randomly at a bus stop at which the bus arrives promptly on the hour and on the half hour.

$\frac{1}{2}$

5- Correctly guessing a stranger's birth date knowing only that she was born on an even-numbered day of April.

$\frac{1}{15}$

7B Combining Probabilities

Determine whether the following events are independent or dependent. Then find the probability of the event.

6- Discovering that your three best friends all have a birthday in April.

$\frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$ or 0.06%

7- Drawing Three B buttons in the first three selections in a game bingo.

$\frac{15}{75} \cdot \frac{14}{74} \cdot \frac{13}{73} = \frac{2730}{405,150} = \frac{91}{13,505}$ or 0.67%

Determine whether the following events are overlapping or non-overlapping. Then find the probability of the event.

8- Getting a sum of either 6 or 8 on a roll of two dice.

$\frac{10}{36} = \frac{5}{18}$ or 27.8%

9- Randomly meeting three international students in a row on a campus where 1 in 12 students is an international student.

$\frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$ or 0.06%

Use the method of your choice to determine the following probabilities:

10- Rolling a 3 or a red number on a die on which the even numbers are red and the odd numbers are black.

$\frac{1}{6} + \frac{3}{6} = \frac{2}{3}$ or 66.7%

11- Being dealt four red cards off the top of a standard deck of well-shuffled cards.

$\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} = \frac{46}{833}$ or 5.5%

12- Randomly meeting either a man or an American in a group composed of 20 English women, 15 English men, 10 American women, and 5 American men.

$\frac{20}{50} + \frac{15}{50} - \frac{5}{50} = \frac{3}{5}$ or 60%

7C The Law of Large Numbers

Find the expected value (to you) of the described game.

13- You are given 7 to 1 odds against rolling a double number (for example, two 1's or two 2's) with the roll of two fair dice, meaning you win \$7 if you succeed and you lose \$1 if you fail.

$(7 \times \frac{1}{6}) + (-1 \times \frac{5}{6}) = \frac{2}{6}$ or $.33$ $\quad 33¢$

Find the expected value (to the company) per policy sold. If the company sells 10,000 policies, what is the expected profit or loss? Explain

14- An insurance policy sells for \$600. Based on past data, an average of 1 in 50 policyholders will file a \$5,000 claim, an average of 1 in 100 policyholders will file a \$10,000 claim, and an average of 1 in 200 policyholders will file a \$30,000 claim.

$(600 \times 1) + (-5{,}000 \times \frac{1}{50}) + (-10{,}000 \times \frac{1}{100}) + (-30{,}000 \times \frac{1}{200}) = \$250$

Profit: $250 \times 10{,}000 = 2.5$ million

15- Averaged over all games and all bets being played, the house edge of a particular casino is \$0.055 per dollar gambled. If a total of \$100 million is wagered in the casino over the course of the year, what is the casino's total profit? Explain

$.055 \times 100$ million $= 5.5$ million

16- When you bet \$5 on the number 7 in roulette at a typical casino, you have a 37/38 probability of losing \$5 and 1/38 probability of making a net gain of \$175 (after paying your bet). If you bet \$5 on an odd number, the probability of losing \$5 is 20/38 and the probability of a net gain of \$5 is 18/38.

a) What is the expected value of betting \$5 on the number 7?

$(-5 \times \frac{37}{38}) + (175 \times \frac{1}{38}) = -0.26$

b) What is the expected value of betting \$5 on an odd number?

$(-5 \times \frac{20}{38}) + (5 \times \frac{18}{38}) = -0.26$

7E Counting and Probability

Answer the following questions using the appropriate counting technique, which may be either arrangements with repetition, permutations, or combinations. Be sure to explain why this counting technique applies to the problem.

17- A city council with eight members must elect a three-person executive committee consisting of a mayor, secretary, and treasurer. How many executive committees are possible?

${}_8P_3 = 336$

18- How many 6- person lineups can be formed from a 12-player volleyball roster, assuming every player can be assigned to any position?

${}_{12}P_6 = 665{,}280$ (order matters)

Find the probability of the given event.

19- Choosing five lottery numbers that match five randomly selected balls when the balls are numbered 1 through 42.

$$\frac{1}{{}_{42}C_5} = \frac{1}{850{,}668}$$

20- Randomly selecting 3 Ohio students from a group of 16 students, 7 of whom are from Ohio.

$$\frac{{}_7C_3}{{}_{16}C_3} = \frac{35}{560} = \frac{1}{16}$$

8A Growth: Linear versus Exponential

State whether the growth (or decay) is linear or exponential, and answer the associated question.

21- The population of Winesburg is increasing at a rate of 3% per year. If the population is 100,000 today, what will it be in three years?

exponential $100{,}000(1+.03)^3 = 109{,}273$

22- The price of a gallon of gasoline is increasing \$0.04 per week. If the price is \$3.10 per gallon today, what will it be in ten weeks?

linear $.04(10) + 3.10 = \$3.50$

8B Doubling Time and Half-Life

Each exercise gives a doubling time for an exponentially growing quantity. Answer the questions that follow.

23- The doubling time of a bank account balance is 30 years.

By what factor does it grow in 60 years? In 90 years?

$2^2 = 4$ $2^3 = 8$

24- The number of cells in a tumor doubles every 6 months. If the tumor begins with a single cell, how many cells will there be after 6 years? After 10 years?

$Q = Q_0 \times 2^{\frac{t}{T_D}}$ $1 \times 2^{\frac{6}{.5}} = 4{,}096$

$1 \times 2^{\frac{10}{.5}} = 1{,}048{,}576$

Use the approximate doubling time formula (rule of 70). Discuss whether the formula is valid for the case described.

25- Oil consumption is increasing at a rate of 2.2% per year. What is its doubling time? By what factor will oil consumption increase in a decade?

Doubling: $T_D = \frac{70}{2.2} \approx 32$ years $2^{\frac{10}{32}} \approx 1.24$

Each exercise gives a half-life for an exponentially decaying quantity. Answer the questions that follow.

26- The current population of a threatened animal species is 1 million, but it is declining with a half-life of 25 years. How many animals will be left in 30 years? In 70 years?

new value $= \text{initial} \times \left(\frac{1}{2}\right)^{\frac{t}{T_{half}}}$

$= 1 \text{ million} \left(\frac{1}{2}\right)^{\frac{30}{25}} \approx 435{,}000$

$1 \text{ million} \left(\frac{1}{2}\right)^{\frac{70}{25}} \approx 144{,}000$

Use the approximate half-life formula. Discuss whether the formula is valid for the case described.

27- A clean-up project is reducing the concentration of a pollutant in the water supply, with a 3.5% decrease per week. What is the approximate half-life of the concentration of the pollutant? About what fraction of the original amount of the pollutant will remain when the project ends after 1 year (52 weeks)?'

$T_{half} = \frac{70}{3.5} = 20$ weeks

$\frac{1}{2}^{\frac{52}{20}} \approx 0.16$

yes it's valid because the percent is less than 15

8C Real Population Growth

Starting from a 2013 population of 7.1 billion, use the given growth rate to find the approximate doubling time (use the rule of 70) and to predict world population in 2050.

28- Use the average annual growth rate between 1950 and 2000, which was about 1.8%

$T_{d} = \frac{70}{1.8} \approx 39$ years

$Q = 7.1 \times (2)^{\frac{37}{39}} \approx 13.7$ billion

29- Starting from the 7.1 billion world population in 2013, assume that world population maintains its current annual growth rate of 0.9%. What will be the world population when you are 50 years old? 80 years old? 100 years old? (Assume you are 17 years old ☺)

$Q = 7.1(1.009)^{t}$

100 = 14.9 billion

50 = 9.5 billion

80 = 12.5 billion

For the given carrying capacities, use a 1960 annual growth rate of 2.1% and population of 3 billion to predict the base growth rate and current growth rate with a logistic model. Assume a current world population of 7.1 billion. How do the predicted growth rates compare to the actual growth rate of about 1.1% per year?

30- Assume the carrying capacity is 20 billion.

Base: $r = \frac{.021}{1 - \frac{3}{20}} \approx 2.47\%$

Current: $.0247\left(1 - \frac{7.1}{20}\right) \approx 1.8\%$

(much higher)

8D Logarithmic Scales: Earthquake, Sounds, Acids

Use the earthquake magnitude scale to answer the questions

31- How many times as much energy is released by an earthquake of magnitude 5 as one by a magnitude 3?

10^{3} more or 1,000 times

mag 3: $2.5 \times 10^{4} \times 10^{1.5(3)} = 2.5 \times 10^{8.5}$

mag 5: $2.5 \times 10^{4} \times 10^{1.5(5)} = 2.5 \times 10^{11.5}$

32- How much energy, in joules, was released by the 2008 earthquake in Sichuan province, China (magnitude 7.9), which killed at least 68,000 people, including many school children?

$2.5 \times 10^{4} \times 10^{1.5(7.9)} \approx 1.8 \times 10^{16}$ joules

9A Functions: The Building Blocks of Mathematical Models

33- The following data table represents a function.

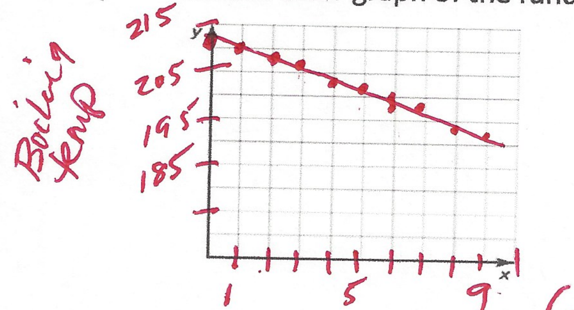
Altitude (ft)	Boiling Point of Water (°F)
0	212.0
1000	210.2
2000	208.4
3000	206.6
4000	204.8
5000	203.0
6000	201.0
7000	199.3
8000	195.5
9000	193.6

independent : altitude
dependent : Boiling temp.

a) Identify the independent and dependent variables, and describe the domain and range.

D: $0 \le a \le 900$ R: $193.6 \le t \le 212$

b) Make a clear graph of the function. Describe the function in words.



the boiling point of water decreases with altitude

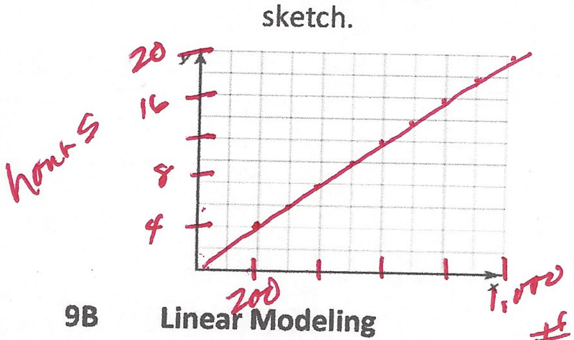
(altitude in 1,000)

34- (number of pages in a book, time to read the book) for a single person.

a) Describe an **appropriate** domain and range for the function.

D: $(30 \le p \le 1{,}000)$ R: $(1 \le h \le 20)$

b) Make a rough sketch of a graph of the function. Explain the assumptions that go into your sketch.



The reading time should increase steadily with length of book

9B Linear Modeling

The following situations can be modeled by linear functions. In each case, write an equation for the linear function and use it to answer the given question. Be sure you clearly identify the independent and dependent variables. Then briefly discuss whether a linear model is reasonable for the situation described.

35- In 2012, Dana Vollmer set the women's world record in the 100-meter butterfly (swimming) with a time of 55.98 seconds. Assume that the record falls at a constant rate of 0.05 seconds per year. What does the model predict for the record in 2020?

independent: time in years let $t=0$ for 2012
dependent: record time (R)
Equation $R = 55.98 - .05t$ in 2020 record is 55.98 sec

36- The cost of leasing a car is \$1,000 for a down payment and processing fee plus \$360 per month. For how many months can you lease a car with \$3,680?

independent is time measured in months.
dependent is amount paid (P) measured in \$
Equation: $P = 1{,}000 + 360t$
$3680 = 1{,}000 + 360t$

$t \simeq 7.4$ months

Create the required linear function, and use it to answer the following questions.

37- A mining company can extract 2,000 tons of gold ore per day with a purity of 3 ounces of gold per ton. The cost of extraction is \$1,000 per ton. If p is the price of gold in dollars per ounce, find a function that gives the daily profit/loss of the mine as it varies with the price of gold. What is the minimum price of gold that makes the mine profitable?

$P = -2{,}000{,}000 + 6{,}000p$

with (0) profit the price is \$333.33

The following situations can be modeled by linear functions. In each case, draw a graph of the function and use the graph to answer the given question. Be sure you clearly identify the independent and dependent variables. Then briefly discuss whether a linear model is reasonable for the situation described.

38- The maximum speed of a semitrailer truck up a steep hill varies with the weight of its cargo. With no cargo, it can maintain a maximum speed of 50 miles per hour. With 20 tons of cargo, its maximum speed drops to 50 miles per hour. At what load does a linear model predict a maximum speed of 0 miles per hour? Variables are (load, max speed) = (l, s)

$(0, 50)$

$(20, 40)$

$s = -\frac{1}{2}l + 50$

when $s = 0$ load is 100

9C Exponential Modeling

39- Consider the following cases of exponential growth and decay.

The number of restaurants in a city that had 800 restaurants in 2013 increases at a rate of 3% per year.

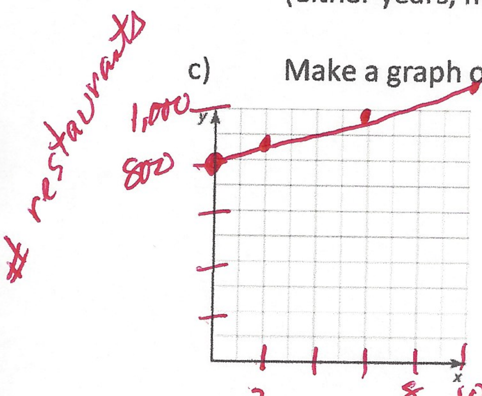
a) Create an exponential function of the form $Q = Q_0 \times (1 + r)^t$ (where $r > 0$ for growth and $r < 0$ for decay) to model the situation described. Be sure to clearly identify both variables in your function.

$Q = 800 \times (1.03)^t$

b) Create a table showing the value of the quantity Q for at least 4 units of time (either years, months, weeks, or hours) of growth or decay.

0	800
2	849
6	955
8	1013
10	1075

c) Make a graph of the exponential function



40- A small city had a population of 110,000 in 2010. Concerned about rapid growth, the residents passed a growth control ordinance limiting population growth to 2% each year. If the population grows at this rate, what will the population be in 2020?

$Q = Q_0 \times (1 + r)^t$

$= 110{,}000 \times (1.02)^{10} \approx 134{,}100$

41- Assume that for the average individual, aspirin has a half-life of 8 hours in the bloodstream. At 12:00 noon, you take a 300-milligram dose of aspirin.

How much aspirin will be in your blood at 6:00 p.m. the same day? At midnight? At noon the next day.

$Q = Q\left(\frac{1}{2}\right)^{\frac{t}{T_{half}}} = 300\left(\frac{1}{2}\right)^{\frac{t}{8}}$

6 p.m about 178.4 mg

midnight about 106.1 mg.

noon nextday about 37.5 mg.